Integrovanie substitučnou metódou – pokračovanie

$$\int \sin(ax+b)dx = \frac{1}{a}\int \sin t\,dt = -\frac{1}{a}\cos t = -\frac{1}{a}\cos(ax+b)+C$$

$$\int cos(ax+b)dx = \frac{1}{a}\int cos\,t\,dt = \frac{1}{a}sin\,t = \frac{1}{a}sin(ax+b)+C$$

$$\int \frac{1}{cos^2(ax+b)}dx = \frac{1}{a}\int \frac{1}{cos^2 t}dt = \frac{1}{a}tg\ t+C = \frac{1}{a}tg(ax+b)+C$$

$$\int \frac{1}{sin^2(ax+b)}dx = \frac{1}{a}\int \frac{1}{sin^2 t}dt = -\frac{1}{a}cot\,g\ t+C = -\frac{1}{a}cot\,g(ax+b)+C$$

Príklad: a) $\int sin(2x-3)dx$, b) $\int sin\,5x.dx$, c) $\int sin\frac{x}{3}dx$, d) $\int sin(1-2x)dx$, e) $\int sin\left(4-\frac{x}{5}\right)dx$,

f) $\int\left(2+sin\frac{x}{4}\right)dx$, g) $\int cos\frac{x}{2}dx$, h) $\int(3-cos\,4x)dx$, i) $\int cos\left(\frac{1}{3}-\frac{x}{3}\right)dx$, j) $\int\frac{1}{cos^2(3x-7)}dx$,

k) $\int\frac{1}{cos^2(8x)}dx$, l) $\int\frac{4}{sin^2(4-5x)}dx$, m) $\int cos^2\,x.dx$, n) $\int sin^2\,x.dx$, o) $\int\frac{1}{1-cos\,x}dx$, p) $\int\frac{1}{1+cos\,x}dx$,

r) $\int\frac{1}{1-cos\,4x}dx$.

$$\int e^{(ax+b)}dx\frac{1}{a}\int e^t dt = \frac{1}{a}e^t + C = \frac{1}{a}e^{(ax+b)}+C$$

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Príklad: a) $\int e^{8x-5}dx$, b) $\int e^{5-7x}dx$, c) $\int e^{-x}dx$, d) $\int e^{3-2x}dx$, e) $\int e^{\frac{x}{3}}dx$, f) $\int 10^{3x+5}dx$, g) $\int 5^{2x}dx$,

h) $\int\frac{e^{2x}-1}{e^x}dx$, i) $\int\left(e^{\frac{x}{2}}+e^{-\frac{x}{2}}\right)dx$, j) $\int\left(e^x+1\right)^3dx$.

$$\int\frac{1}{\sqrt{(ax+b)^2+A}}dx = \frac{1}{a}\int\frac{1}{\sqrt{t^2+A}}dt = \frac{1}{a}ln\left|t+\sqrt{t^2+A}\right|+C = \frac{1}{a}ln\left|(ax+b)+\sqrt{(ax+b)^2+A}\right|+C$$

$\int\frac{1}{\sqrt{ax^2+bx+c}}dx$, *pre a⟩0*, odmocnenca upravujeme doplnením na štvorec dvojčlena a uvedením na predchádzajúci tvar

Príklad: $\int\frac{1}{\sqrt{x^2+2x+5}}dx = \int\frac{1}{\sqrt{(x+1)^2+4}}dx = ln\left|(x+1)+\sqrt{(x+1)^2+4}\right|+C$.

Príklad: $\int\frac{1}{\sqrt{x^2-4x+10}}dx = \int\frac{1}{\sqrt{(x-2)^2+6}}dx = ln\left|(x-2)+\sqrt{(x-2)^2+6}\right|+C$.

Vypočítajte príklady: a) $\int \frac{1}{\sqrt{1+(x-1)^2}}dx$, b) $\int \frac{1}{\sqrt{(3x-5)^2+1}}dx$, c) $\int \frac{1}{\sqrt{x^2+4x+11}}dx$,

d) $\int \frac{1}{\sqrt{2x^2-x+2}}dx$, e) $\int \frac{2}{\sqrt{4x^2+4x+3}}dx$.

$$\int \frac{a.f'(x)}{f(x)}dx = \left| \begin{matrix} s: t = f(x) \\ dt = f'(x)dx \end{matrix} \right| = a\int \frac{dt}{t} = a.ln|t| + C = a.ln|f(x)| + C$$

Príklad. a) $\int \frac{tgx}{cos^2 x}dx$, b) $\int \frac{cot\, gx}{sin^2 x}dx$, c) $\int \frac{1}{x.ln\, x}dx$, d) $\int \frac{4x-8}{x^2-8x+7}dx$, e) $\int \frac{x}{x^2+3}dx$, f) $\int \frac{sin\, x}{cos\, x}dx$,

g) $\int tgx.dx$, h) $\int cot\, gx.dx$, i) $\int \frac{e^x}{e^x+4}dx$, j) $\int \frac{sin\, 2x}{3+sin^2 x}dx$.