Vypočítajte limity:

11.) $\lim\limits_{x \to \pi} \frac{\sin 2x + 2\sin x}{\sin^2 x}$ 12.) $\lim\limits_{x \to \pi} \frac{\cos^2 x - \cos 2x}{1 + \cos x}$

13.) $\lim\limits_{x \to \frac{\pi}{2}} \frac{2\cos x - \sin 2x}{\cos^2 x}$ 14.) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cos 2x + \sin^2 x}{1 - \sin x}$

Riešenie:

11.) $$\lim_{x \to \pi} \frac{\sin 2x + 2\sin x}{\sin^2 x} = \lim_{x \to \pi} \frac{2\sin x . \cos x + 2\sin x}{1 - \cos^2 x} = \lim_{x \to \pi} \frac{2\sin x(\cos x + 1)}{(1 - \cos x)(1 + \cos x)} =$$
$$= \lim_{x \to \pi} \frac{2\sin x}{1 - \cos x} = \frac{2\sin \pi}{1 - \cos \pi} = \frac{0}{2} = 0$$

12.) $$\lim_{x \to \pi} \frac{\cos^2 x - \cos 2x}{1 + \cos x} = \lim_{x \to \pi} \frac{\cos^2 x - \cos^2 x + \sin^2 x}{1 + \cos x} = \lim_{x \to \pi} \frac{\sin^2 x}{1 + \cos x} =$$
$$= \lim_{x \to \pi} \frac{(1 + \cos x)(1 - \cos x)}{1 + \cos x} = \lim_{x \to \pi} (1 - \cos x) = 1 - \cos \pi = 1 - (-1) = 1 + 1 = 2$$

13.) $$\lim_{x \to \frac{\pi}{2}} \frac{2\cos x - \sin 2x}{\cos^2 x} = \lim_{x \to \frac{\pi}{2}} \frac{2\cos x - 2\sin x . \cos x}{(1 - \sin x)(1 + \sin x)} = \lim_{x \to \frac{\pi}{2}} \frac{2\cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} =$$
$$= \lim_{x \to \frac{\pi}{2}} \frac{2\cos x}{1 + \sin x} = \frac{2\cos \frac{\pi}{2}}{1 + \sin \frac{\pi}{2}} = \frac{0}{2} = 0$$

14.) $$\lim_{x \to \frac{\pi}{2}} \frac{\cos 2x + \sin^2 x}{1 - \sin x} = \lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x - \sin^2 x + \sin^2 x}{1 - \sin x} = \lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x} =$$
$$= \lim_{x \to \frac{\pi}{2}} \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x} = \lim_{x \to \frac{\pi}{2}} (1 + \sin x) = 1 + \sin \frac{\pi}{2} = 1 + 1 = 2$$