

Vypočítajte limity:

$$\begin{array}{ll} 7.) \quad {}_x\lim_{-2} \frac{3x+6}{x^3+8} & 8.) \quad {}_x\lim_0 \frac{\sqrt{2+x}-\sqrt{2}}{x} \\ 9.) \quad {}_x\lim_0 \left[\frac{\sin x}{x} + \frac{\sin 3x}{x} + \frac{\sin 5x}{x} \right] & 10.) \quad {}_x\lim_0 \frac{\sin x}{\sqrt{1-\cos x}} \end{array}$$

Riešenie:

$$\begin{aligned} 7.) \quad {}_x\lim_{-2} \frac{3x+6}{x^3+8} &= {}_x\lim_{-2} \frac{3(x+2)}{(x+2)(x^2-2x+4)} = {}_x\lim_{-2} \frac{3}{x^2-2x+4} = \\ &= \frac{3}{(-2)^2-2(-2)+4} = \frac{3}{12} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 8.) \quad {}_x\lim_0 \frac{\sqrt{2+x}-\sqrt{2}}{x} &= {}_x\lim_0 \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} = {}_x\lim_0 \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} = \\ &= {}_x\lim_0 \frac{1}{\sqrt{2+x}+\sqrt{2}} = \frac{1}{\sqrt{2+0}+\sqrt{2}} = \frac{1}{2\sqrt{2}} \end{aligned}$$

$$9.) \quad {}_x\lim_0 \left[\frac{\sin x}{x} + \frac{\sin 3x}{x} + \frac{\sin 5x}{x} \right] = {}_x\lim_0 \left[\frac{\sin x}{x} + \frac{3\sin 3x}{3x} + \frac{5\sin 5x}{5x} \right] = 1 + 3 \cdot 1 + 5 \cdot 1 = 9$$

$$\begin{aligned} 10.) \quad {}_x\lim_0 \frac{\sin x}{\sqrt{1-\cos x}} &= {}_x\lim_0 \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = {}_x\lim_0 \sqrt{\frac{1-\cos^2 x}{1-\cos x}} = {}_x\lim_0 \sqrt{\frac{(1-\cos x)(1+\cos x)}{1-\cos x}} = \\ &= {}_x\lim_0 \sqrt{1+\cos x} = \sqrt{1+\cos 0} = \sqrt{1+1} = \sqrt{2} \end{aligned}$$