Určitý integrál - substitúcia

Riešenie:

1.) $\int_0^1 \frac{1}{(5x+1)^3}dx = \int_1^6 t^{-3}\frac{dt}{5} = \frac{1}{5}\int_1^6 t^{-3}dt = \left[-\frac{t^{-2}}{10}\right]_1^6 = \left(-\frac{1}{10.36}+\frac{1}{10.1}\right) =$ $\quad t = 5x+1 \Rightarrow dx = \frac{dt}{5}$

$= -\frac{1}{360}+\frac{1}{10} = \frac{35}{360} = \frac{7}{72}$ $\quad t_1 = 5.0+1 = 1$

$t_2 = 5.1+1 = 6$

2.) $\int_0^2 x\sqrt{4-x^2}dx = \int_4^0 x\sqrt{t}\frac{dt}{-2x} = -\frac{1}{2}\int_4^0 t^{\frac{1}{2}}dt = +\frac{1}{2}\int_0^4 t^{\frac{1}{2}}dt = \frac{1}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 =$ $\quad t = 4-x^2 \Rightarrow dx = \frac{dt}{-2x}$

$= \frac{1}{2}.\frac{2}{3}\left[\sqrt{t^3}\right]_0^4 = \frac{1}{3}\left(\sqrt{4^3}-\sqrt{0}\right) = \frac{1}{3}.8 = \frac{8}{3}$ $\quad t_1 = 4-0^2 = 4$

$t_2 = 4-2^2 = 0$

3.) $\int_3^5 \sqrt{x-3}dx = \int_0^2 t^{\frac{1}{2}}dt = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 = \left[\frac{2}{3}\sqrt{t^3}\right]_0^2 = \frac{2}{3}\left(\sqrt{2^3}-\sqrt{0^3}\right) = \frac{2}{3}\sqrt{8} =$ $\quad t = x-3 \Rightarrow dx = dt$

$= \frac{2.2,\sqrt{2}}{3} = \frac{4}{3}\sqrt{2},$ $\quad t_1 = 3-3 = 0$

$t_2 = 5-3 = 2$

4.) $\int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}dx = \int_4^1 \frac{x}{\sqrt{t}}\cdot\frac{dt}{-2x} = -\frac{1}{2}t^{-\frac{1}{2}}dt = +\frac{1}{2}\int_1^4 t^{-\frac{1}{2}}dt = \frac{1}{2}\left[2.t^{\frac{1}{2}}\right]_1^4 =$ $\quad t = 4-x^2 \Rightarrow dx = \frac{dt}{-2x}$

$= \left[\sqrt{t}\right]_1^4 = \sqrt{4}-\sqrt{1} = 2-1 = 1,$ $\quad t_1 = 4-0^2 = 4$

$t_2 = 4-\left(\sqrt{3}\right)^2 = 1$