Určitý integrál - Leibnitz a Newtonova metóda

Riešenie:

2.) $\int_2^5 (2x+3)dx = [2x^2+3x]_2^5 = (2.5^2+3.5)-(2.2^2+3.2) = 65-14 = 51,$

3.) $\int_1^3 (3x^2-2x+1)dx = [x^3-x^2+x]_1^3 = (3^3-3^2+3)-(1^3-1^2+1) = 21-1 = 20$

4.) $\int_0^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1-0 = 1$

5.) $\int_1^3 \frac{1}{1+x}dx = [\ln|1+x|]_1^3 = \ln|1+3| - \ln|1+1| = \ln|4| - \ln|2| = \ln\left|\frac{4}{2}\right| = \ln|2|$

6.) $\int_0^{\frac{\pi}{4}} \frac{1}{2\cos^2 x}dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x}dx = \frac{1}{2}[tgx]_0^{\frac{\pi}{4}} = \frac{1}{2}\left(tg\frac{\pi}{4} - tg0\right) = \frac{1}{2}(1-0) = \frac{1}{2}$

7.) $\int_0^1 \frac{e^x}{1+e^x}dx = \left[\ln\left|1+e^x\right|\right]_0^1 = \ln\left|1+e^1\right| - \ln\left|1+e^0\right| = \ln\left|1+e\right| - \ln\left|2\right| = \ln\left|\frac{1+e}{2}\right|$

8.) $\int_1^4 \sqrt{x}dx = \int_1^4 x^{\frac{1}{2}}dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \left[\frac{2}{3}\sqrt{x^3}\right]_1^4 = \frac{2}{3}\left(\sqrt{4^3} - \sqrt{1^3}\right) = \frac{2}{3}(8-1) = \frac{2}{3}.7 = \frac{14}{3}$

9.) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\cos x}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin x.\cos x}{\cos x}dx = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\,dx = -2\left[\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -2\left(\cos\frac{\pi}{3} - \cos\frac{\pi}{6}\right) =$

$= -2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \sqrt{3} - 1,$

10.) $\int_0^5 \frac{\cos^4 x - \sin^4 x}{\cos 2x}dx = \int_0^5 \frac{\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right)}{\cos^2 x - \sin^2 x}dx = \int_0^5 \left(\cos^2 x + \sin^2 x\right)dx =$

$= \int_0^5 1.dx = \left[x\right]_0^5 = 5 - 0 = 5$

11.) $\int_0^4 \frac{2x^2 - 50}{x-5}dx = 2\int_0^4 \frac{x^2 - 25}{x-5}dx = 2\int_0^4 \frac{(x-5)(x+5)}{x-5}dx = 2\int_0^4 (x+5)dx = 2\left[\frac{x^2}{2} + 5x\right]_0^4 =$

$= \left[x^2 + 10x\right]_0^4 = \left(4^2 + 10.4\right) - 0 = 16 + 40 = 56,$

12.) $a^2\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = a^2\left[\frac{x}{2} + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}} = a^2\left(\frac{\pi}{4} + \frac{1}{4}\sin\pi - 0 + 0\right) = a^2\left(\frac{\pi}{4} + 0 - 0 + 0\right) = \frac{1}{4}\pi.a^2$

13.) $\int_0^1 \frac{1}{\sqrt{x^2 - 1}}dx =$

Integrál nie je definovaný lebo $x \neq \pm 1 \vee x \neq 0$