Zopakujte si základné vzorce integrovania priamou metódou

Neurčitý integrál:

$$\int f(x)dx = F(x)+C \quad \Rightarrow \quad \left[F(x)+C\right]' = f(x)$$

f(x) dx je integrand, F(x) je primitívna funkcia, C je integračná konštanta

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

2) $\int \frac{1}{x} dx = \ln|x| + C$

3) $\int e^x dx = e^x + C$

4) $\int a^x dx = \frac{a^x}{\ln a} + C$

5) $\int \sin x\, dx = -\cos x + C$

6) $\int \cos x dx = \sin x + C$

7) $\int \frac{1}{\cos^2 x} dx = tg\, x + C$

8) $\int \frac{1}{\sin^2 x} dx = -\cot g\, x + C$

9) $\int \frac{1}{1+x^2} dx = arctg\, x + C = -arc\cot g\, x + C$

10) $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C = -\arccos x + C$

11) $\int \cos^2 x\, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

12) $\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

13) $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$

14) $\int \ln x\, dx = x(\ln|x| - 1) + C$

15) $\int \frac{1}{\sin x} dx = \ln\left|tg\frac{x}{2}\right| + C$

16) $\int \frac{1}{\cos x} dx = \ln\left|tg\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

Riešenie:

2) $\int \frac{(x-1)^2}{\sqrt{x}}dx = \int \frac{(x^2-2x+1)}{x^{\frac{1}{2}}}dx = \int \left(\frac{x^2}{x^{\frac{1}{2}}} - 2\frac{x}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}}\right)dx = \int \left(x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{\frac{-1}{2}}\right)dx =$

$= \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C = \frac{2}{5}\sqrt{x^5} - \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} +$

3) $\int \sqrt{x}\left(1+\sqrt[3]{x}\right)dx = \int x^{\frac{1}{2}}\left(1+x^{\frac{1}{3}}\right)dx = \int \left(x^{\frac{1}{2}} + x^{\frac{5}{6}}\right)dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{6}{11}x^{\frac{11}{6}} + C =$

$= \frac{2}{3}\sqrt{x^3} + \frac{6}{11}\sqrt[6]{x^{11}} + C$

4) $\int \frac{1}{\sqrt[4]{x}}dx = \int x^{-\frac{1}{4}}dx = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$

5) $\int \left(x+\frac{1}{x^2}\right)^2 dx = \int \left(x^2 + \frac{2}{x} + \frac{1}{x^4}\right)dx = \int \left(x^2 + 2\frac{1}{x} + x^{-4}\right)dx = \frac{x^3}{3} + 2\ln|x| - \frac{1}{3x^3} + C$

6) $\int \left(\frac{\sin 2x}{\cos x}\right)dx = \int \frac{2\sin x.\cos x}{\cos x}dx = 2.\int \sin x\,dx = -2\cos x + C$

7) $\int \left(\sin^2 x + \cos^2 x\right)dx = \int 1dx = x + C$

8) $\int \frac{1}{\sin^2 x.\cos^2 x}dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x.\cos^2 x}dx = \int \left(\frac{\sin^2 x}{\sin^2 x.\cos^2 x} + \frac{\cos^2 x}{\sin^2 x.\cos^2 x}\right)dx =$

$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)dx = tgx - \cot gx + C$

9) $\int \frac{\cos 2x}{\sin^2 x.\cos^2 x}dx = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x.\cos^2 x}dx = \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\right)dx = C - \cot g\,x - tg\,x$

10.) $\int \frac{1+\sin^2 x}{1-\cos^2 x}dx = \int \frac{1+\sin^2 x}{\sin^2 x}dx = \int \left(\frac{1}{\sin^2 x}+1\right)dx = C - \cot g\, x + x$

11) $\int tg^2 x\, dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx = \int \left(\frac{1}{\cos^2 x}-1\right)dx = tg\, x - x + C$

12) $\int \frac{\cos 2x}{\sin x+\cos x}dx = \int \frac{\cos^2 x-\sin^2 x}{\sin x+\cos x}dx = \int \frac{(\cos x+\sin x)(\cos x-\sin x)}{\sin x+\cos x}dx =$

$= \int (\cos x-\sin x)dx = \sin x+\cos x+C$

13) $\int \sqrt{1+\sin 2x}\, dx = \int \sqrt{\sin^2 x+\cos^2 x+2\sin x.\cos x}\, dx = \int \sqrt{(\sin x+\cos x)^2}\, dx =$

$= \int (\sin x+\cos x)dx = C-\cos x+\sin x$

14.) $\int \frac{1}{1+\cos 2x}dx = \int \frac{1}{\sin^2 x+\cos^2 x+\cos^2 x-\sin^2 x}dx = \frac{1}{2}\int \frac{1}{\cos^2 x}dx = \frac{1}{2}tg\, x + C$

15.) $\int \frac{1}{1-\cos 2x}dx = \int \frac{1}{\sin^2 x+\cos^2 x-\cos^2 x+\sin^2 x}dx = \frac{1}{2}\int \frac{1}{\sin^2 x}dx = -\frac{1}{2}\cot g\, x + C$

16.) $\int \frac{\cos^4 x-\sin^4 x}{\cos 2x}dx = \int \frac{(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)}{\cos^2 x-\sin^2 x}dx = \int (\cos^2 x+\sin^2 x)dx =$

$= \frac{x}{2}+\frac{1}{4}\sin 2x+\frac{x}{2}-\frac{1}{4}\sin 2x+C = 2.\frac{x}{2}+C = x+C$

17.) $\int \frac{3-2\cot g^2 x}{\cos^2 x}dx = \int (\frac{3}{\cos^2 x}-\frac{2\cot g^2 x}{\cos^2 x})dx = \int \left(\frac{3}{\cos^2 x}-\frac{2}{\sin^2 x}\right)dx =$

$= 3tg\, x + 2\cot g\, x + C$

18.) $\int \frac{18x^2-2}{3x-1}dx = 2.\int \frac{9x^2-1}{3x-1}dx = 2\int \frac{(3x-1)(3x+1)}{3x-1}dx = 2\int (3x+1)dx =$

$= 2\left[\frac{3x^2}{2}+x\right]+C = 3x^2+2x+C$

19.) $\int \frac{4-x}{2+\sqrt{x}}dx = \int \frac{(2+\sqrt{x})(2-\sqrt{x})}{2+\sqrt{x}}dx = \int (2-\sqrt{x})dx = \int \left(2-x^{\frac{1}{2}}\right)dx =$

$= 2x - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C = 2x-\frac{2}{3}\sqrt{x^3}+C$

20.) $\int \frac{3x^4-48}{2x^2+8}dx = \frac{3}{2}\int \frac{x^4-16}{x^2+4}dx = \frac{3}{2}\int \frac{(x^2+4)(x^2-4)}{x^2+4}dx =$

$= \frac{3}{2}\int (x^2-4)dx = \frac{3}{2}\left(\frac{x^3}{3}-4x+C\right) = \frac{x^3}{2}-6x+C$

21.) $\int \left(\frac{2}{1+x^2}-\frac{3}{\sqrt{1-x^2}}\right)dx = 2\int \frac{1}{1+x^2}dx - 3\int \frac{1}{\sqrt{1-x^2}}dx =$

$= 2arctg\, x - 3\arcsin x + C$

22.) $\int \frac{2x}{x^2+2008}dx = \ln\left|x^2+2008\right|+C$

23.) $\int 3.2^x dx = 3\int 2^x dx = 3.\frac{2^x}{\ln 2}+C = \frac{3.2^x}{\ln 2}+C$

24.) $\int \frac{x^2}{x^3-1939}dx = \frac{1}{3}\int \frac{3x^2}{x^3-1939}dx = \frac{1}{3}.\ln\left|x^3-1939\right|+C$

25.) $\int \frac{\sin x}{13+\cos x}dx = (-1)\int \frac{(-1)\sin x}{13+\cos x}dx = -\ln\left|13+\cos x\right|+C$

26.) $\int \frac{e^x}{e^x+\pi}dx = \ln\left|e^x+\pi\right|+C$

27.) $\int tg\,x\,dx = \int \frac{\sin\ x}{\cos x}dx = (-1)\int \frac{(-\sin\ x)}{\cos\ x}dx = -\ln\left|\cos x + C\right|$

28.) $\int \cot gx\,dx = \int \frac{\cos x}{\sin\ x}dx = \ln\left|\sin\ x\right| + C$

29.) $\int e^x\left(1-\frac{e^{-x}}{x}\right)dx = \int\left(e^x - \frac{e^x.e^{-x}}{x}\right)dx = \int\left(e^x - \frac{1}{x}\right)dx = e^x - \ln\left|x\right| + C$

30.) $\int \frac{1+\cos x}{x+\sin x}dx = \ln\left|x+\sin\ x\right| + C$

31.) $\int \frac{\frac{1}{x}}{17+\ln\ x}dx = \ln\left|17+\ln\left|x\right|\right| + C$

32.) $\int \frac{x+1}{x-1}dx = \int\left(1+\frac{2}{x-1}\right)dx = x + 2.\ln\left|x-1\right| + C$

33.) $\int \frac{3x-5}{x+2}dx = \int\left(3-\frac{11}{x+2}\right)dx = 3x - 11.\ln\left|x+2\right| + C$

34.) $\int \frac{x^2-5x+7}{x-1}dx = \int\left(x-4+\frac{3}{x-1}\right)dx = \frac{x^2}{2} - 4x + 3.\ln\left|x-1\right| + C$

35.) $\int \frac{2x^3-3x^2+5x-4}{x-2}dx = \int\left(2x^2+x+7+\frac{10}{x-2}\right)dx =$

$= \frac{2x^3}{3} + \frac{x^2}{2} + 7x + 10.\ln\left|x-2\right| + C$