Derivácia zloženej funkcie

1 Derivujte a upravte funkcie:

1.) $y = \left(x^3 - 2\right)^5$ 2.) $y = \dfrac{1}{(5-2x)^2}$

3.) $y = \sqrt[3]{x^3 - 3}$ 4.) $y = \sqrt{8 - \dfrac{1}{x^2}}$

Riešenie:

1.) $y = \left(x^3 - 2\right)^5$

$$y' = \left[\left(x^3 - 2\right)^5\right]'\left(x^3 - 2\right)' = 5\left(x^3 - 2\right)^4 3x^2 = 15x^2\left(x^3 - 2\right)^4$$

2.) $y = \dfrac{1}{(5-2x)^2} = (5-2x)^{-2}$

$$y' = \left[(5-2x)^{-2}\right]'(5-2x)' = -2(5-2x)^{-3}(-2) = \frac{4}{(5-2x)^3}$$

3.) $y = \sqrt[3]{x^3 - 3} = \left(x^3 - 3\right)^{\frac{1}{3}}$

$$y' = \left[\left(x^3 - 3\right)^{\frac{1}{3}}\right]'\left(x^3 - 3\right)' = \frac{1}{3}\left(x^3 - 3\right)^{-\frac{2}{3}} 3x^2 = \frac{x^2}{\sqrt[3]{\left(x^3 - 3\right)^2}}$$

4.) $y = \sqrt{8 - \dfrac{1}{x^2}} = \left(8 - x^{-2}\right)^{\frac{1}{2}}$

$$y' = \left[\left(8 - x^{-2}\right)^{\frac{1}{2}}\right]'\left(8 - x^{-2}\right)' = \frac{1}{2}\left(8 - x^{-2}\right)^{-\frac{1}{2}}\left(2x^{-3}\right) = \frac{1}{x^3 . \sqrt{8 - \dfrac{1}{x^2}}}$$

2 Derivujte a upravte funkcie:

5.) $y = \sin 2x$ 6.) $y = \sin\left(\frac{x}{3}+\frac{\pi}{2}\right)$ 7.) $y = \sqrt{1+\sin x}$

8.) $y = \sin^2 x$ 9.) $y = \sin^2 x.\cos^2 x$

Riešenie:

5.) $y = \sin 2x$

$$y' = [\sin 2x]'(2x)' = \cos 2x.2 = 2.\cos 2x$$

6.) $y = \sin\left(\frac{x}{3}+\frac{\pi}{2}\right)$

$$y' = \left[\sin\left(\frac{x}{3}+\frac{\pi}{2}\right)\right]'\left(\frac{x}{3}+\frac{\pi}{2}\right)' = \cos\left(\frac{x}{3}+\frac{\pi}{2}\right).\frac{1}{3} = \frac{1}{3}\cos\left(\frac{x}{3}+\frac{\pi}{2}\right)$$

7.) $y = \sqrt{1+\sin x} = (1+\sin x)^{\frac{1}{2}}$

$$y' = \left[(1+\sin x)^{\frac{1}{2}}\right]'(1+\sin x)' = \frac{1}{2}(1+\sin x)^{-\frac{1}{2}}.\cos x = \frac{\cos x}{2\sqrt{1+\sin x}}$$

8.) $y = \sin^2 x$

$$y' = [\sin^2 x]'.(\sin x)' = 2\sin x.\cos x = \sin 2x$$

9.) $y = \sin^2 x.\cos^2 x$

$$y' = [\sin^2 x]'.\cos^2 x + \sin^2 x[\cos^2 x]' = 2\sin x.\cos x.\cos^2 x + \sin^2 x.(-2\cos x.\sin x) =$$

$$= 2\sin x.\cos^3 x - 2\cos x.\sin^3 x = 2\sin x.\cos x.(\cos^2 x - \sin^2 x) = \sin 2x.\cos 2x.$$

3 Derivujte a upravte funkcie:

10.) $y = e^{\frac{x}{2}}$ 11.) $y = e^{1+\cos x}$

12.) $y = e^{\sqrt{x}}$ 13.) $y = \sin\left(e^x + \pi\right)$

Riešenie:

10.) $y = e^{\frac{x}{2}}$

$$y' = \left[e^{\frac{x}{2}}\right]' \left(\frac{x}{2}\right)' = e^{\frac{x}{2}} . \frac{1}{2} = \frac{1}{2} e^{\frac{x}{2}}$$

11.) $y = e^{1+\cos x}$

$$y' = \left[e^{1+\cos x}\right]' (1 + \cos x)' = \left[e^{1+\cos x}\right] (-\sin x) = -\sin x . e^{1+\cos x}$$

12.) $y = e^{\sqrt{x}} = e^{x^{\frac{1}{2}}}$

$$y' = \left[e^{x^{\frac{1}{2}}}\right]' \left(x^{\frac{1}{2}}\right)' = e^{x^{\frac{1}{2}}} . \frac{1}{2} x^{-\frac{1}{2}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

13.) $y = \sin\left(e^x + \pi\right)$

$$y' = \left[\sin\left(e^x + \pi\right)\right]' \left(e^x + \pi\right)' = \cos\left(e^x + \pi\right) e^x = e^x \cos\left(e^x + \pi\right)$$

4 Derivujte a upravte funkcie:

14.) $y=\sqrt{\dfrac{1+e^x}{1-e^x}}$ 15.) $y=\sqrt{\dfrac{e^x-e^{-x}}{e^x+e^{-x}}}$ 16.) $y=e^x\sqrt{1+x^2}$

Riešenie:

14.) $y=\sqrt{\dfrac{1+e^x}{1-e^x}}=\left(\dfrac{1+e^x}{1-e^x}\right)^{\frac{1}{2}}$

$$y'=\left[\left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}}\right]'\left(\frac{1+e^x}{1-e^x}\right)'=$$

$$=\frac{1}{2}\left(\frac{1+e^x}{1-e^x}\right)^{-\frac{1}{2}}\frac{(1+e^x)'(1-e^x)-(1+e^x)(1-e^x)'}{(1-e^x)^2}=$$

$$=\frac{1}{2}\left(\frac{1+e^x}{1-e^x}\right)^{-\frac{1}{2}}\frac{2e^x}{(1-e^x)^2}=\frac{e^x}{\sqrt{\dfrac{1+e^x}{1-e^x}(1-e^x)^4}}=$$

$$=\frac{e^x}{\sqrt{(1+e^x)(1-e^x)(1-e^x)^2}}=$$

$$=\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$$

15.) $y=\sqrt{\dfrac{e^x-e^{-x}}{e^x+e^{-x}}}=\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)^{\frac{1}{2}}$

$$y'=\left[\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)^{\frac{1}{2}}\right]'\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)'=\frac{1}{2}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)^{-\frac{1}{2}}\frac{4}{(e^x+e^{-x})^2}=$$

$$=\frac{2}{(e^x+e^{-x})\sqrt{e^{2x}-e^{2x}}}$$

16.) $y=e^x\sqrt{1+x^2}=e^x(1+x^2)^{\frac{1}{2}}$

5 Derivujte a upravte funkcie:

17.) $y = \ln\left(7 + x + x^2\right)$ 18.) $y = \ln(\sin x) - \ln(\cos x)$

19.) $y = \ln \frac{x^2 - 5}{x^2 + 5}$

Riešenie:

17.) $y = \ln\left(7 + x + x^2\right)$

$$y' = \left[\ln\left(7 + x + x^2\right)\right]'\left(7 + x + x^2\right)' = \frac{1}{7 + x + x^2}(1 + 2x) = \frac{1 + 2x}{7 + x + x^2}$$

18.) $y = \ln(\sin x) - \ln(\cos x) = \ln \frac{\sin x}{\cos x} = \ln tgx$

$$y' = [\ln tgx]'(tgx)' = \frac{1}{tgx} \cdot \frac{1}{\cos^2 x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} =$$

$$= \frac{1}{\sin x . \cos x} = \frac{2}{2 \sin x . \cos x} = \frac{2}{\sin 2x}$$

19.) $y = \ln \frac{x^2 - 5}{x^2 + 5}$

$$y' = \left[\ln \frac{x^2 - 5}{x^2 + 5}\right]'\left(\frac{x^2 - 5}{x^2 + 5}\right)' =$$

$$= \frac{1}{\frac{x^2 - 5}{x^2 + 5}} \frac{\left(x^2 - 5\right)'\left(x^2 + 5\right) - \left(x^2 - 5\right)\left(x^2 + 5\right)'}{\left(x^2 + 5\right)^2} =$$

$$= \frac{x^2 + 5}{x^2 - 5} \cdot \frac{2x\left(x^2 + 5\right) - 2x\left(x^2 - 5\right)}{\left(x^2 + 5\right)^2} = \frac{1}{\left(x^2 - 5\right)\left(x^2 + 5\right)} \cdot (20x) = \frac{20x}{x^4 - 25}$$

6 Derivujte a upravte funkcie:

$$20.)\quad y=\ln\frac{1+e^x}{1-e^x}\qquad 21.)\quad y=\ln\frac{1+\sin x}{1-\sin x}$$

$$22.)\quad y=\ln\left(x+\sqrt{1+x^2}\right)$$

Riešenie:

$$20.)\quad y=\ln\frac{1+e^x}{1-e^x}$$

$$y'=\left[\ln\frac{1+e^x}{1-e^x}\right]'\left(\frac{1+e^x}{1-e^x}\right)'=\frac{1}{\dfrac{1+e^x}{1-e^x}}\cdot\frac{\left(1+e^x\right)'\left(1-e^x\right)-\left(1+e^x\right)\left(1-e^x\right)'}{\left(1-e^x\right)^2}=$$

$$=\frac{1-e^x}{1+e^x}\,\frac{2e^x}{\left(1-e^x\right)^2}=\frac{2e^x}{\left(1+e^x\right)\left(1-e^x\right)}=\frac{2e^x}{1-e^{2x}}$$

$$21.)\quad y=\ln\frac{1+\sin x}{1-\sin x}$$

$$y'=\left[\ln\frac{1+\sin x}{1-\sin x}\right]'\left(\frac{1+\sin x}{1-\sin x}\right)'=$$

$$=\frac{1}{\dfrac{1+\sin x}{1-\sin x}}\cdot\frac{\left(1+\sin x\right)'\left(1-\sin x\right)-\left(1+\sin x\right)\left(1-\sin x\right)'}{\left(1-\sin x\right)^2}=$$

$$=\frac{1-\sin x}{1+\sin x}\cdot\frac{2\cos x}{\left(1-\sin x\right)^2}=\frac{2\cos x}{\left(1+\sin x\right)\left(1-\sin x\right)}=\frac{2\cos x}{1-\sin^2 x}=\frac{2\cos x}{\cos^2 x}=\frac{2}{\cos x}$$

$$22.)\quad y=\ln\left(x+\sqrt{1+x^2}\right)=\ln\left[x+\left(1+x^2\right)^{\frac{1}{2}}\right]$$

$$y'=\left(\ln\left(x+\sqrt{1+x^2}\right)\right)'=\frac{1}{\left(x+\sqrt{1+x^2}\right)}\left[1+\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}.2x\right]=$$

$$=\frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{x}{\sqrt{1+x^2}}\right)=$$

$$=\frac{1}{x+\sqrt{1+x^2}}\cdot\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}=\frac{1}{\sqrt{1+x^2}}$$

7 Derivujte a upravte funkcie:

23.) $y = \ln\left(5e^x + x^5\right)$ 24.) $y = \ln\left[tg\left(\frac{x}{2}\right)\right]$ 25.) $y = \frac{1}{8}\ln\frac{x^8-1}{x^8+1}$

Riešenie:

23.) $y = \ln\left(5e^x + x^5\right)$

$$y' = \left[\ln\left(5e^x + x^5\right)\right]'\left(5e^x + x^5\right)' = \frac{1}{5e^x + x^5}\left(5e^x + 5x^4\right) = \frac{5.\left(e^x + x^4\right)}{5e^x + x^5}$$

24.) $y = \ln\left[tg\left(\frac{x}{2}\right)\right]$

$$y' = \left\{\ln\left[tg\left(\frac{x}{2}\right)\right]\right\}' . .\left[tg\left(\frac{x}{2}\right)\right]' .\left(\frac{x}{2}\right)' = \frac{1}{tg\left(\frac{x}{2}\right)}\frac{1}{\cos^2\left(\frac{x}{2}\right)}\frac{1}{2} = \frac{1}{\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}.\cos^2\frac{x}{2}}.\frac{1}{2} =$$

$$= \frac{1}{2\sin\frac{x}{2}.\cos\frac{x}{2}} = \frac{1}{\sin\left(2.\frac{x}{2}\right)} = \frac{1}{\sin x}$$

25.) $y = \frac{1}{8}\ln\frac{x^8-1}{x^8+1}$

$$y' = \frac{1}{8}\left[\ln\frac{x^8-1}{x^8+1}\right]'\left(\frac{x^8-1}{x^8+1}\right)' = \frac{1}{8}.\frac{x^8+1}{x^8-1}\frac{\left(x^8-1\right)'\left(x^8+1\right) - \left(x^8-1\right)\left(x^8+1\right)'}{\left(x^8+1\right)^2} =$$

$$= \frac{1}{8\left(x^8-1\right)\left(x^8+1\right)}\left[8x^7\left(x^8+1\right) - 8x^7\left(x^8-1\right)\right] = \frac{16x^7}{8\left(x^{16}-1\right)} = \frac{2x^7}{x^{16}-1}$$

8 Derivujte a upravte funkcie:

26.) $y = arc\ \cos\frac{x}{5}$ 27.) $y = arc\ \sin\sqrt{x}$ 28.) $y = arc\,tg\frac{2x}{1-x^2}$

Riešenie:

26.) $y = arc\ \cos\frac{x}{5}$

$$y' = \left[arc\ \cos\frac{x}{5}\right]'\left(\frac{x}{5}\right)' = -\frac{1}{\sqrt{1-\left(\frac{x}{5}\right)^2}}.\frac{1}{5} = -\frac{\frac{1}{5}}{\sqrt{\frac{25-x^2}{25}}} = -\frac{\frac{1}{5}}{\frac{\sqrt{25-x^2}}{5}} = -\frac{1}{\sqrt{25-x^2}}$$

27.) $y = arc\ \sin\sqrt{x} = arc\ \sin(x)^{\frac{1}{2}}$

$$y' = \left[arc\ \sin(x)^{\frac{1}{2}}\right]'\left((x)^{\frac{1}{2}}\right)' = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}.\frac{1}{2}.x^{-\frac{1}{2}} = \frac{1}{2.\sqrt{x}.\sqrt{1-x}}$$

28.) $y = arc\,tg\frac{2x}{1-x^2}$

$$y' = \left[arc\,tg\frac{2x}{1-x^2}\right]'\left(\frac{2x}{1-x^2}\right)' = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}.\frac{(2x)'(1-x^2)-2x(1-x^2)'}{(1-x^2)^2} =$$

$$= \frac{(1-x^2)^2}{1+2x^2+x^4}.\frac{2+2x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1+x^2)^2} = \frac{2}{(1+x^2)}$$