

## Cvičenia

1. Vypočítajte neurčité integrály:

- a)  $\int \frac{1}{4-9x^2} dx$   $\left[ \frac{1}{12} \ln \left| \frac{2+3x}{2-3x} \right| + C \right]$
- b)  $\int \frac{1}{x+x^2} dx$   $\left[ \ln \left| \frac{x}{x+1} \right| + C \right]$
- c)  $\int \frac{1}{(x+1)(x-2)} dx$   $\left[ \frac{1}{3} \ln \left| \frac{x-2}{x+1} \right| + C \right]$
- d)  $\int \frac{1}{x^2+5x+11} dx$   $\left[ 2\sqrt{19} \operatorname{arctg} \frac{2x+5}{\sqrt{19}} + C \right]$
- e)  $\int \frac{1}{x^2-4x+12} dx$   $\left[ \frac{1}{2\sqrt{2}} \operatorname{arctg} \frac{x-2}{2\sqrt{2}} + C \right]$
- f)  $\int \frac{x}{(1+x^2)^3} dx$   $\left[ -\frac{1}{4(1+x^2)^2} + C \right]$
- g)  $\int x \sqrt[3]{x+2} dx$   $\left[ \frac{3}{7} \sqrt[3]{(x+2)^7} - \frac{3}{2} \sqrt[3]{(x+2)^4} + C \right]$
- h)  $\int x \sqrt{1-x^2} dx$   $\left[ -\frac{1}{3} \sqrt{(1-x^2)^3} + C \right]$
- i)  $\int \frac{1}{\sqrt{9-4x^2}} dx$   $\left[ \frac{1}{2} \arcsin \frac{2x}{3} + C \right]$
- j)  $\int \frac{1}{\sqrt{1-3x^2}} dx$   $\left[ \frac{1}{\sqrt{3}} \arcsin x\sqrt{3} + C \right]$
- k)  $\int \frac{1}{\sqrt{x(1-x)}} dx$   $\left[ 2 \arcsin \sqrt{x} + C \right]$
- l)  $\int \frac{1}{\sqrt{x^2+4x+5}} dx$   $\left[ \ln(x+2+\sqrt{x^2+4x+5}) + C \right]$
- m)  $\int \frac{4^x}{1+4^{2x}} dx$   $\left[ \frac{1}{\ln 4} \operatorname{arctg} 4^x + C \right]$
- n)  $\int \frac{3^x}{\sqrt{1-9^x}} dx$   $\left[ \frac{\arcsin 3^x}{\ln 3} + C \right]$
- o)  $\int \frac{\ln^4 x}{x} dx$   $\left[ \frac{1}{5} \ln^5 x + C \right]$
- p)  $\int e^x \cot ge^x dx$   $\left[ \ln |\sin e^x| + C \right]$
- q)  $\int \sin^3 x \cos x dx$   $\left[ \frac{1}{4} \sin^4 x + C \right]$
- r)  $\int \cos^2 x \sin x dx$   $\left[ -\frac{1}{3} \cos^3 x + C \right]$

$$\begin{aligned} \text{s) } \int \frac{\sin x}{\sqrt{\cos^5 x}} dx & \left[ \frac{2}{3\sqrt{\cos^3 x}} + C \right] \\ \text{t) } \int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx & \left[ 3\sqrt[3]{\sin x} + C \right] \end{aligned}$$

2. Vypočítajte neurčité integrály:

$$\begin{aligned} \text{a) } \int \cos 3x \sin 4x dx & \left[ -\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + C \right] \\ \text{b) } \int \cos 2x \cos 6x dx & \left[ \frac{1}{8} \sin 4x + \frac{1}{16} \sin 8x + C \right] \\ \text{c) } \int \sin x \sin 3x dx & \left[ \frac{1}{4} \sin 2x - \frac{1}{8} \sin 4x + C \right] \\ \text{d) } \int \sin x \sin 2x \sin 3x dx & \left[ -\frac{1}{8} \left( \frac{\cos 4x}{2} + \cos 2x - \frac{\cos 6x}{3} \right) + C \right] \\ \text{e) } \int \frac{x - \operatorname{arctg} x}{1+x^2} dx & \left[ \frac{1}{2} \ln(1+x^2) - \frac{1}{2} \operatorname{arctg}^2 x + C \right] \\ \text{f) } \int \frac{e^x \sqrt{\operatorname{arctg} e^x}}{1+e^{2x}} dx & \left[ \frac{2}{3} \sqrt{\operatorname{arctg}^3 e^x} + C \right] \\ \text{g) } \int x e^{2x} dx & \left[ \frac{e^{2x}(2x-1)}{4} + C \right] \\ \text{h) } \int x \sin x dx & \left[ \sin x - x \cos x + C \right] \\ \text{i) } \int x \ln x dx & \left[ \frac{x^2(2 \ln x - 1)}{4} + C \right] \end{aligned}$$

3. Vypočítajte neurčité integrály:

$$\begin{aligned} \text{a) } \int x \ln^2 x dx & \left[ \frac{x^2(2 \ln^2 x - 2 \ln x + 1)}{4} + C \right] \\ \text{b) } \int x \operatorname{arctg} x dx & \left[ \frac{1}{2} (x^2 + 1) \operatorname{arctg} x - \frac{x}{2} + C \right] \\ \text{c) } \int x^2 e^{-x} dx & \left[ -e^{-x}(x^2 + 2x + 2) + C \right] \\ \text{d) } \int x^2 \ln x dx & \left[ \frac{1}{3} x^3 \left( \ln x - \frac{1}{3} \right) + C \right] \\ \text{e) } \int x^2 \sin 2x dx & \left[ -\frac{1}{4} (2x^2 - 1) \cos 2x + \frac{1}{2} x \sin 2x + C \right] \\ \text{f) } \int x^2 \operatorname{arctg} x dx & \left[ \frac{1}{3} x^3 \operatorname{arctg} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2) + C \right] \\ \text{g) } \int \ln x dx & \left[ x \ln x - x + C \right] \\ \text{h) } \int \operatorname{arctg} x dx & \left[ x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C \right] \\ \text{i) } \int \arcsin x dx & \left[ x \arcsin x + \sqrt{1-x^2} + C \right] \end{aligned}$$

j) $\int \ln(x + \sqrt{1+x^2}) dx$	$[x \ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + C]$
k) $\int \operatorname{arctg} \sqrt{x} dx$	$[x \operatorname{arctg} \sqrt{x} - \sqrt{x} + \operatorname{arctg} \sqrt{x} + C]$
l) $\int \frac{x e^x}{(1+x)^2} dx$	$\left[ \frac{e^x}{x+1} + C \right]$
m) $\int \frac{x \arcsin x}{\sqrt{1-x^2}} dx$	$[x - \sqrt{1-x^2} \arcsin x + C]$
n) $\int e^x \sin x dx$	$\left[ \frac{1}{2} e^x (\sin x - \cos x) + C \right]$
o) $\int e^{2x} \cos x dx$	$\left[ \frac{1}{5} e^{2x} (\sin x + 2 \cos x) + C \right]$

4. Vypočítajte neurčité integrály:

a) $\int \frac{5x^3 + 9x^2 - 22x - 8}{x^3 - 4x} dx$	$[5x + 2 \ln x  + 3 \ln x-2  + 4 \ln x+2  + C]$
b) $\int \frac{9x^4 + 3x^3 - 23x^2 + x}{9x^3 - 6x^2 - 5x + 2} dx$	$\left[ \frac{1}{2} x^2 + x - \frac{2}{3} \ln 3x+2  + \frac{1}{3} \ln 3x-1  - \ln x-1  + C \right]$
c) $\int \frac{9x-14}{9x^2-24x+16} dx$	$\left[ \frac{2}{9x-12} + \ln 3x-4  + C \right]$
d) $\int \frac{3x-4}{(x-2)(x-1)^3} dx$	$\left[ \frac{4x-5}{2(x-1)^2} + 2 \ln \left  \frac{x-2}{x-1} \right  + C \right]$
e) $\int \frac{x^2 + 3x + 2}{x^2 + x + 2} dx$	$\left[ x + \ln x^2 + x + 2  - \frac{2}{\sqrt{7}} \operatorname{arctg} \frac{2x+1}{\sqrt{7}} + C \right]$
f) $\int \frac{x^3 + x - 1}{x(x^2 + 1)} dx$	$\left[ x + \ln \frac{\sqrt{x^2+1}}{ x } + C \right]$
g) $\int \frac{1}{x^3 + 1} dx$	$\left[ \frac{1}{6} \ln \frac{(x+1)^2}{x^2 - x + 1} + \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{2x-1}{\sqrt{3}} + C \right]$
h) $\int \frac{1}{x^3 + x^2 + x} dx$	$\left[ \frac{1}{2} \ln \frac{x^2}{x^2 + x + 1} - \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} + C \right]$

5. Vypočítajte neurčité integrály:

a) $\int \frac{x}{x + \sqrt{x}} dx$	$[x - 2\sqrt{x} + 2 \ln(\sqrt{x} + 1) + C]$
b) $\int \frac{1 - \sqrt{x}}{1 + \sqrt{x}} dx$	$[-x + 4\sqrt{x} - 4 \ln(\sqrt{x} + 1) + C]$
c) $\int \frac{\sqrt[3]{x}}{x + \sqrt[6]{x^5}} dx$	$[3\sqrt[3]{x} - 6\sqrt[6]{x} + 6 \ln(\sqrt[6]{x} + 1) + C]$
d) $\int \frac{\sqrt[6]{x} + 1}{\sqrt[6]{x^7} + \sqrt[4]{x^5}} dx$	$\left[ -\frac{6}{\sqrt[6]{x}} + \frac{12}{\sqrt[12]{x}} + 24 \ln \left  \sqrt[12]{x} (\sqrt[12]{x} + 1) \right  + C \right]$

e)  $\int \sqrt{\frac{1+x}{1-x}} dx$   $\left[ \arcsin x - \sqrt{1-x^2} + C \right]$

f)  $\int \sqrt[3]{\frac{1-x}{1+x}} dx$   $\left[ \frac{1}{3} \ln \left| \frac{u^2 + u + 1}{(u-1)^3} \right| + \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2u+1}{\sqrt{3}} + \frac{2u}{u^3-1} + C, \text{ kde } u = \sqrt[3]{\frac{1-x}{1+x}} \right]$

g)  $\int \sqrt{\frac{1+x}{1-x}} \frac{1}{(1-x)(1+x)^2} dx$   $\left[ \frac{x}{\sqrt{1-x^2}} + C \right]$

h)  $\int \frac{1}{\sqrt{x^2 + x + 1}} dx$   $\left[ \ln \left| \sqrt{1+x+x^2} + x + \frac{1}{2} \right| + C \right]$

i)  $\int \frac{x}{\sqrt{3x^2 - 5x + 8}} dx$   $\left[ \frac{1}{\sqrt{3}} \ln \left| x\sqrt{3} - 5\frac{\sqrt{3}}{6} + \sqrt{3x^2 - 5x + 8} \right| + C \right]$

j)  $\int \frac{1}{\sqrt{3-2x-5x^2}} dx$   $\left[ \frac{1}{\sqrt{5}} \arcsin \frac{5x+1}{4} + C \right]$

k)  $\int \frac{x}{\sqrt{x^2 - 4x + 1}} dx$   $\left[ \sqrt{x^2 - 4x + 1} + 2 \ln \left| 2x - 4 + 2\sqrt{x^2 - 4x + 1} \right| + C \right]$

l)  $\int \frac{1}{x\sqrt{3+2x+x^2}} dx$   $\left[ -\frac{1}{\sqrt{3}} \ln \frac{3+x}{3x} \sqrt{\frac{x^2+2x+3}{3x^2}} + C, \text{ pre } x > 0 \right]$

m)  $\int \frac{1}{(x-1)\sqrt{x^2 - x - 1}} dx$   $\left[ \arcsin \frac{x-3}{\sqrt{5}(x-1)} + C, \text{ pre } x > 1 \right]$

n)  $\int \frac{2}{x - \sqrt{x^2 - 1}} dx$   $\left[ x^2 + x\sqrt{x^2 + 1} + \ln \left| x + \sqrt{x^2 + 1} \right| + C \right]$

o)  $\int \frac{1}{x - \sqrt{x^2 - x + 1}} dx$   $\left[ 2 \ln |t| - \frac{3}{2} \ln |2t + 1| + \frac{3}{2(2t+1)} + C, \text{ kde } t = \sqrt{x^2 - x + 1} - x \right]$

p)  $\int \frac{x^5}{\sqrt{x^2 + 1}} dx$   $\left[ \left( \frac{x^4}{5} - \frac{4x^2}{15} + \frac{8}{15} \right) \sqrt{1+x^2} + C \right]$

q)  $\int \frac{x^6}{\sqrt{1-x^2}} dx$   $\left[ \left( -\frac{x^5}{6} - \frac{5x^3}{24} - \frac{5x}{16} \right) \sqrt{1-x^2} + \frac{5}{16} \arcsin x + C \right]$

r)  $\int \sqrt{3+4x+x^2} dx$   $\left[ \frac{x+2}{2} \sqrt{3+4x+x^2} - \frac{1}{2} \ln \left| x+2 + \sqrt{3+4x+x^2} \right| + C \right]$

s)  $\int \sqrt{3-2x-x^2} dx$   $\left[ \frac{x+2}{2} \sqrt{3-2x-x^2} + 2 \arcsin \frac{x+1}{2} + C \right]$

6. Vypočítajte neurčité integrály:

a)  $\int \frac{1}{4-5 \sin x} dx$   $\left[ \frac{1}{3} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 2}{2 \operatorname{tg} \frac{x}{2} - 1} \right| + C \right]$

b)	$\int \frac{1}{3-5\cos x} dx$	$\left[ \frac{1}{4} \ln \left  \frac{2\operatorname{tg} \frac{x}{2} - 1}{2\operatorname{tg} \frac{x}{2} + 1} \right  + C \right]$
c)	$\int \frac{1}{\sin x - \cos x} dx$	$\left[ \frac{1}{\sqrt{2}} \ln \left  \operatorname{tg} \frac{\pi}{8} - \frac{x}{2} \right  + C \right]$
d)	$\int \frac{1}{\cos x - 2\sin x + 3} dx$	$\left[ \arctg \left( \operatorname{tg} \frac{x}{2} - 1 \right) + C \right]$
e)	$\int \frac{1-\operatorname{tg} x}{1+\operatorname{tg} x} dx$	$[\ln \sin x + \cos x  + C]$
f)	$\int \frac{1}{4-3\sin^2 x} dx$	$\left[ \frac{1}{2} \operatorname{arctg} \frac{\operatorname{tg} x}{2} + C \right]$
g)	$\int \cos^5 x dx$	$\left[ \sin x - \frac{2}{3} \sin^3 x + \frac{\sin^5 x}{5} + C \right]$
h)	$\int \sin^6 3x dx$	$\left[ \frac{5x}{16} - \frac{\sin 6x}{12} + \frac{\sin 12x}{64} - \frac{\sin^3 6x}{144} + C \right]$
i)	$\int \cos^2 x \sin x dx$	$\left[ \sin x - \frac{2}{3} \sin^3 x + \frac{\sin^5 x}{5} + C \right]$
j)	$\int \cos^2 x \sin^2 x dx$	$\left[ \frac{x}{8} - \frac{\sin 4x}{32} + C \right]$
k)	$\int \cos^2 x \sin^3 x dx$	$\left[ -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C \right]$
l)	$\int \sin^5 \frac{x}{2} \cos^3 \frac{x}{2} dx$	$\left[ \frac{1}{4} \sin^8 \frac{x}{2} - \frac{1}{3} \sin^6 \frac{x}{2} + C \right]$
m)	$\int \sin^4 x \cos^4 x dx$	$\left[ \frac{1}{128} \left( \frac{1}{8} \sin 8x - \sin 4x - 3x \right) + C \right]$
n)	$\int \frac{1}{\cos x} dx$	$\left[ \ln \left  \operatorname{tg} \left( \frac{\pi}{4} + \frac{x}{2} \right) \right  + C \right]$
o)	$\int \frac{1}{\sin x \cos^2 x} dx$	$\left[ \ln \left  \operatorname{tg} \frac{\pi}{2} \right  + \frac{1}{\cos x} + C \right]$
p)	$\int \frac{1}{\sin^3 x \cos^5 x} dx$	$\left[ -\frac{\cot g^2 x}{2} - \ln \cot gx  + \frac{3}{2} \cot g^2 x + \frac{1}{4} \cot g^4 x + C \right]$
q)	$\int \frac{1}{\sin^4 x \cos^2 x} dx$	$\left[ \operatorname{tg} x - 2 \cot gx - \frac{\cot g^3 x}{3} + C \right]$
r)	$\int \sin 3x \sin 5x dx$	$\left[ -\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C \right]$
s)	$\int \cos 3x \cos 4x dx$	$\left[ \frac{\sin x}{2} + \frac{\sin 7x}{14} + C \right]$
t)	$\int \sin x \sin 2x \sin 3x dx$	$\left[ -\frac{\cos 2x}{8} - \frac{\cos 4x}{16} + \frac{\cos 6x}{24} + C \right]$