1) Dané sú komplexné čísla a = 1+2i , b = 2 – i. Určite a + b, a – b, a.b, a / b, |a|

Zistite či číslo $a = \frac{4}{5} - i\frac{3}{5}$ *je komplexná jednotka*

Riešenie:

a) a + b = (1+2i) + (2- i) = 3+i
b) a – b = (1+2i) – (2 – i) = –1 + 3i
c) a .b = (1 + 2i)(2 – i) =2 +4i -1 -2i² =2 +3i +2 = 4 +3i

$$d)\ \frac{1+2i}{2-i} = \frac{1+2i}{2-i}\cdot\frac{2+i}{2+i} = \frac{2+4i+i+2i^2}{4-i^2} = \frac{2+5i-2}{4+1} = \frac{5i}{5} = i$$

$$e)\ |a| = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$$

$$a = \frac{4}{5} - i\frac{3}{5}$$

$$|a| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}+\frac{9}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$

Číslo a je komplexná jednotka.

a) a = 4 – 3i b = 2 + i
b) a = -2 + 3i b = 2 – 5i
c) a = 5 – 3i b = -4 -2i
d) a = -2i b = - 7 – i

a) a = -2 -2i
b) a = 1+i
c) $a = -\frac{3}{5} - \frac{4}{5}i$

2) Vypočítajte:

$$\left|\frac{1-2i}{3+4i}\right|$$

Riešenie:

$$\frac{1-2i}{3+4i}\cdot\frac{3-4i}{3-4i} = \frac{3-6i-4i+8i^2}{9-16i^2} = \frac{-5-10i}{9+16} = \frac{5(-1-2i)}{25} = \frac{-1-2i}{5} = -\frac{1}{5} - \frac{2}{5}i$$

$$\left|\frac{1-2i}{3+4i}\right| = \sqrt{\left(-\frac{1}{5}\right)^2 + \left(-\frac{2}{5}\right)^2} = \sqrt{\frac{1}{25}+\frac{4}{25}} = \sqrt{\frac{5}{25}} = \frac{\sqrt{5}}{5},$$

a) $\left|\frac{3-2i}{5-4i}\right|$
b) $\left|\frac{-2-3i}{5-4i}\right|$
c) $\left|\frac{-4+3i}{-5i}\right|$

d) $\left|\frac{-6+i}{3-8i}\right|$

3) Vypočítajte:

$$X=\left(\frac{1-i}{1+i}\right)^2-\left(\frac{1+i}{1-i}\right)^2$$

Riešenie:

$$X=\left(\frac{1-i}{1+i}\right)^2-\left(\frac{1+i}{1-i}\right)^2$$

$$A=\left(\frac{1-i}{1+i}\right)^2=\frac{1-2i+i^2}{1+2i+i^2}=\frac{1-2i-1}{1+2i-1}=-\frac{2i}{2i}=-1$$

$$B=\left(\frac{1+i}{1-i}\right)^2=\frac{1+2i+i^2}{1-2i+i^2}=\frac{1+2i-1}{1-2i-1}=-\frac{2i}{2i}=-1$$

$$X=A-B$$

$$X=-1-(-1)=-1+1=0$$

$$X=\left(\frac{1-i}{1+i}\right)^2-\left(\frac{1+i}{1-i}\right)^2=0$$

a) $X=\left(\frac{2-3i}{1-5i}\right)^2+\left(\frac{3-2i}{-3i}\right)^2$

b) $X=\left(\frac{4-5i}{-3-2i}\right)^2-\left(\frac{2-7i}{5+4i}\right)^2$

c) $X=\left(\frac{2-9i}{4-i}\right)^2-\left(\frac{5+4i}{4-3i}\right)^2$