

Limita funkcie

Príklady na precvičovanie

1. Vypočítajte limity funkcií (dosadením za x)

- 1.01 $\lim_{x \rightarrow 2} \frac{x^2 - 9}{x + 2}$ $[-1, 25]$
- 1.02 $\lim_{x \rightarrow -2} \frac{2x^2 + 1}{x + 1}$ $[-9]$
- 1.03 $\lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$, ak sa x postupne blíži k 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ $\left[\frac{1}{2}, 2(2 - \sqrt{3}), 2 - \sqrt{2}, \frac{2}{3}, 1\right]$
- 1.04 $\lim_{x \rightarrow 0} \frac{1}{1 + \sin x}$, ak sa x postupne blíži k 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ [zprava dolava 1.03]
- 1.05 $\lim_{x \rightarrow 0} \frac{\sin 2x}{\cos x}$, ak sa x postupne blíži k 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ $[0, 1, \sqrt{2}, \sqrt{3}]$
- 1.06 $\lim_{x \rightarrow 0} \frac{\cos 2x}{\cos^2 x}$, ak sa x postupne blíži k 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ $\left[1, \frac{2}{3}, 0, -2\right]$
- 1.07 $\lim_{x \rightarrow \frac{\pi}{6}} x^{\sin x}$ $\left[\sqrt{\frac{\pi}{6}}\right]$
- 1.08 $\lim_{x \rightarrow \frac{\pi}{3}} x^{\cos x}$ $\left[\sqrt{\frac{\pi}{3}}\right]$
- 1.09 $\lim_{x \rightarrow \frac{\pi}{4}} (\log \sin x)$ $[-0,5 \log 2]$
- 1.10 $\lim_{x \rightarrow \frac{\pi}{4}} (\log \cos x)$ $[-0,5 \log 2]$
- 1.11 $\lim_{x \rightarrow \log 100} (x^2 2^x)$ $[16]$
- 1.12 $\lim_{x \rightarrow \log 0,01} (x^2 2^x)$ $[1]$
- 1.13 $\lim_{x \rightarrow 1000} \log(x^{\log x})$ $[9]$
- 1.14 $\lim_{x \rightarrow 1000} (\log x)^{\log x}$ $[27]$

2. Vypočítajte limity v bodoch nespojitosti (rozkladom a krátením)

2.01	$\lim_{x \rightarrow 1,5} \frac{4x^2 - 9}{2x - 3}$	$[6]$
2.02	$\lim_{x \rightarrow -2} \frac{x^4 - 16}{x + 2}$	$[-32]$
2.03	$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x} - 1}$	$[4]$
2.04	$\lim_{x \rightarrow -2} \frac{x^2 + 2x}{3x^2 - 12}$	$\left[\frac{1}{6}\right]$
2.05	$\lim_{x \rightarrow 5} \frac{x^3 - 25x}{2x - 10}$	$[25]$
2.06	$\lim_{x \rightarrow -2} \frac{x^4 - 16}{8x + 16}$	$[-4]$
2.07	$\lim_{x \rightarrow 3} \frac{2x - 6}{3x^2 - 27}$	$\left[\frac{1}{9}\right]$
2.08	$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^4 - 1}$	$\left[\frac{1}{2}\right]$
2.09	$\lim_{x \rightarrow 1} \frac{x^5 - x}{x^3 - x}$	$[2]$
2.10	$\lim_{x \rightarrow 1} \frac{x^5 - x^3}{x^4 - 1}$	$\left[\frac{1}{2}\right]$
2.11	$\lim_{x \rightarrow 0,5} \frac{4x^3 - x}{2x - 1}$	$[1]$
2.12	$\lim_{x \rightarrow 2} \frac{x^3 - 8}{4 - x^2}$	$[-3]$
2.13	$\lim_{x \rightarrow 2} \frac{6x - 3x^2}{8 - x^3}$	$\left[\frac{1}{2}\right]$
2.14	$\lim_{x \rightarrow 3} \frac{2x^2 - 6x}{x^3 - 27}$	$\left[\frac{2}{9}\right]$
2.15	$\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - x - 2}$	$\left[-\frac{1}{3}\right]$
2.16	$\lim_{x \rightarrow -4} \frac{x^2 - 3x - 28}{x^2 - 16}$	$\left[\frac{11}{8}\right]$

2.17	$\lim_{x \rightarrow 1} \frac{x^3 - x}{5x^2 + 10x - 15}$	$\left[\frac{1}{10} \right]$
2.18	$\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 + x - 12}$	$\left[\frac{8}{7} \right]$
2.19	$\lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x^2 - 5x + 4}$	$\left[\frac{2}{3} \right]$
2.20	$\lim_{x \rightarrow 4} \frac{x^2 + 3x - 28}{2x - 8}$	$\left[\frac{11}{2} \right]$
2.21	$\lim_{x \rightarrow 5} \frac{x^3 - 25x}{2x^2 - 12x + 10}$	$\left[\frac{25}{4} \right]$
2.22	$\lim_{x \rightarrow 4} \frac{x^2 + 7x - 44}{x^2 - 6x + 8}$	$\left[\frac{15}{2} \right]$
2.23	$\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{3x^2 - 12x}$	$\left[\frac{1}{4} \right]$
2.24	$\lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{2x^2 - 8x}$	$\left[\frac{1}{4} \right]$
2.25	$\lim_{x \rightarrow 1} \frac{x^2 - 6x + 5}{x^3 - x}$	$[-2]$
2.26	$\lim_{x \rightarrow 3} \frac{2x^2 - x - 15}{3x^2 - 8x - 3}$	$\left[\frac{11}{10} \right]$
2.27	$\lim_{x \rightarrow -6} \frac{x^2 + 5x - 6}{x^2 - 36}$	$\left[\frac{7}{12} \right]$
2.28	$\lim_{x \rightarrow 1} \frac{2x^2 - 2}{3x^2 - x - 2}$	$\left[\frac{6}{5} \right]$
2.29	$\lim_{x \rightarrow 3} \frac{-x^2 + 9}{2x^2 - 5x - 3}$	$\left[\frac{6}{7} \right]$
2.30	$\lim_{x \rightarrow -1} \frac{2x^3 + 2x^2 + 3x + 3}{x^3 + x^2 + x + 1}$	$\left[\frac{5}{2} \right]$
2.31	$\lim_{x \rightarrow 1} \frac{2x^2 + x - 3}{3x^2 + 3x - 6}$	$\left[\frac{5}{9} \right]$
2.32	$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\cos^2 x}$	$\left[\frac{1}{2} \right]$

2.33	$\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin^2 x}$	$\left[\frac{1}{2}\right]$
2.34	$\lim_{x \rightarrow \pi} \frac{\sin^2 x}{1 + \cos x}$	$[2]$
2.35	$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\sin x - 1}$	$[-2]$
2.36	$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$	$[\sqrt{2}]$
2.37	$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{\sin^3 x}$	$\left[\frac{1}{2}\right]$
2.38	$\lim_{x \rightarrow 0} \frac{\cos^2 x - \cos 2x}{1 - \cos x}$	$[2]$
2.39	$\lim_{x \rightarrow \frac{3}{2}\pi} \frac{\sin^2 x + \cos 2x}{1 + \sin x}$	$[2]$
2.40	$\lim_{x \rightarrow \frac{3}{2}\pi} \frac{\sin 2x + 2 \cos x}{\cos^2 x}$	$[0]$
2.41	$\lim_{x \rightarrow \frac{\pi}{2}} \frac{2 \cos x - \sin 2x}{\cos^2 x}$	$[0]$
2.42	$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 2x + \sin^2 x}{1 - \sin x}$	$[2]$
2.43	$\lim_{x \rightarrow \pi} \frac{\cos^2 x - \cos 2x}{1 + \cos x}$	$[2]$
2.44	$\lim_{x \rightarrow \pi} \frac{\sin 2x + 2 \sin x}{\sin^2 x}$	$[0]$

3. Vypočítajte limity v bodoch nespojitosti (použitím vzťahu $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$)

3.01	$\lim_{x \rightarrow 0} \frac{\sin^n x}{x^n}$	$[1]$
3.02	$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2}$	$[1]$
3.03	$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x}$	$[0]$

$$3.04 \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \quad [2]$$

$$3.05 \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} \quad [1]$$

$$3.06 \quad \lim_{x \rightarrow 0} x \cot g x \quad [1]$$

$$3.07 \quad \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x \sin x} \quad [2]$$

$$3.08 \quad \lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{x^2} \quad [1]$$

$$3.09 \quad \lim_{x \rightarrow 0} \frac{\sin 4x + \sin 7x}{\sin 3x} \quad \left[\frac{11}{3} \right]$$

Rozšírte vhodným výrazom tak, aby ste mohli použiť vzorec $A^2 - B^2 = (A - B)(A + B)$

$$3.10 \quad \lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x-5} \quad \left[\frac{1}{4} \right]$$

$$3.11 \quad \lim_{x \rightarrow -3} \frac{\sqrt{12+x} - 3}{x+3} \quad \left[\frac{1}{6} \right]$$

$$3.12 \quad \lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{2x} \quad \left[\frac{1}{12} \right]$$

$$3.13 \quad \lim_{x \rightarrow -2} \frac{\sqrt{6+x} - 2}{x+2} \quad \left[\frac{1}{4} \right]$$

$$3.14 \quad \lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \quad \left[\frac{\sqrt{2}}{4} \right]$$

$$3.15 \quad \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1} \quad \left[\frac{2}{3} \right]$$

$$3.16 \quad \lim_{x \rightarrow 0} \frac{x^2 + x}{\sqrt{9+x} - \sqrt{9-x}} \quad [3]$$

$$3.17 \quad \lim_{x \rightarrow 0} \frac{\sqrt{4+x} - \sqrt{4-x}}{x} \quad \left[\frac{1}{2} \right]$$

$$3.18 \quad \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x+1} - \sqrt{2x-1}} \quad [-2\sqrt{3}]$$

$$3.19 \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1-2x}}{x^2 + x} \quad \left[\frac{5}{2} \right]$$

$$\begin{array}{ll}
3.20 & \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{2-x}-\sqrt{6+x}} \quad [-2] \\
3.21 & \lim_{x \rightarrow 2} \frac{\sqrt{6-x}-\sqrt{x+2}}{x-2} \quad \left[-\frac{1}{2}\right] \\
3.22 & \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{7-x}-\sqrt{x+11}} \quad [-3] \\
3.23 & \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-\sqrt{1-2x}} \quad \left[\frac{2}{5}\right] \\
3.24 & \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-\sqrt{7-x}}{x-3} \quad \left[\frac{1}{2}\right] \\
3.25 & \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} \quad [1] \\
3.26 & \lim_{x \rightarrow 2} \frac{\sqrt{3+x+x^2}-\sqrt{9-2x+x^2}}{x^2-3x+2} \quad \left[\frac{1}{2}\right]
\end{array}$$

4. Vypočítajte limity v nevlastnom čísle (kráťte najvyššou mocninou x) a použite $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$

$$\begin{array}{ll}
4.01 & \lim_{x \rightarrow \infty} \frac{x+2}{3x} \quad \left[\frac{1}{3}\right] \\
4.02 & \lim_{x \rightarrow \infty} \frac{5-2x^2}{3x+5x^2} \quad [-0,4] \\
4.03 & \lim_{x \rightarrow \infty} \frac{2x-5}{6-5x^2} \quad [0] \\
4.04 & \lim_{x \rightarrow \infty} \frac{x^2+1}{2x^2+2x-1} \quad [0,5] \\
4.05 & \lim_{x \rightarrow \infty} \frac{2x^3-3x^2+4}{5x-x^2-7x^3} \quad \left[-\frac{2}{7}\right] \\
4.06 & \lim_{x \rightarrow \infty} \frac{2x^2-4x+8}{x^3+2x^2-1} \quad [0] \\
4.07 & \lim_{x \rightarrow \infty} \frac{5x^2-8}{2x^2+5x-1} \quad \left[\frac{5}{2}\right] \\
4.08 & \lim_{x \rightarrow \infty} \frac{2-3x-6x^2}{12x^2+4x-10} \quad [-0,5] \\
4.09 & \lim_{x \rightarrow \infty} \frac{2x^2+3}{x^2+x-1} \quad [2]
\end{array}$$

4.10	$\lim_{x \rightarrow \infty} \frac{3x^3 - 4x + 1}{2x^4 + 5x^2 - x}$	$[0]$
4.11	$\lim_{x \rightarrow \infty} \frac{(x+1)(x+2)(x-1)}{2-x^3}$	$[-1]$
4.12	$\lim_{x \rightarrow \infty} \frac{(2x+1)^2}{x(5x-1)}$	$\left[\frac{4}{5}\right]$

5. Načrtnite graf danej funkcie a odhadnite limitu funkcie z priebehu grafu

5.01	$\lim_{x \rightarrow \infty} 0,99^x$	$[0]$
5.02	$\lim_{x \rightarrow -\infty} 0,99^x$	$[\infty]$
5.03	$\lim_{x \rightarrow \infty} 1,01^x$	$[\infty]$
5.04	$\lim_{x \rightarrow -\infty} 1,01^x$	$[0]$
5.05	$\lim_{x \rightarrow \infty} 2^{x-1}$	$[\infty]$
5.06	$\lim_{x \rightarrow -\infty} 2^{x-1}$	$[0]$
5.07	$\lim_{x \rightarrow \infty} \log_2 x$	$[\infty]$
5.08	$\lim_{x \rightarrow 0^+} \log_2 x$	$[-\infty]$
5.09	$\lim_{x \rightarrow \infty} \log_{\frac{1}{2}} x$	$[-\infty]$
5.10	$\lim_{x \rightarrow 0^+} \log_{\frac{1}{2}} x$	$[\infty]$
5.11	$\lim_{x \rightarrow 0^+} \frac{1}{x}$	$[\infty]$
5.12	$\lim_{x \rightarrow \infty} \frac{1}{x}$	$[0]$
5.13	$\lim_{x \rightarrow 0^-} \frac{1}{x}$	$[-\infty]$
5.14	$\lim_{x \rightarrow -\infty} \frac{1}{x}$	$[0]$
5.15	$\lim_{x \rightarrow 0^+} \frac{-2}{x}$	$[-\infty]$

5.16	$\lim_{x \rightarrow \infty} \frac{-2}{x}$	$[0]$
5.17	$\lim_{x \rightarrow 0^-} \frac{-2}{x}$	$[\infty]$
5.18	$\lim_{x \rightarrow -\infty} \frac{-2}{x}$	$[0]$
5.19	$\lim_{x \rightarrow 0^+} x^{-2}$	$[\infty]$
5.20	$\lim_{x \rightarrow 0^-} x^{-2}$	$[\infty]$
5.21	$\lim_{x \rightarrow \frac{\pi}{2}^-} \operatorname{tg} x$	$[\infty]$
5.22	$\lim_{x \rightarrow \frac{\pi}{2}^+} \operatorname{tg} x$	$[-\infty]$
5.23	$\lim_{x \rightarrow \pi^-} \cot g x$	$[-\infty]$
5.24	$\lim_{x \rightarrow \pi^+} \cot g x$	$[\infty]$