VÝSLEDKY CVIČENÍ

1.1

1. a) $[x_1, -y_1]$; b) $[-x_1, y_1]$. **3.** $B[-2, 3, -1]$.

1.2

1. $[0, -3]$, $[0, -9]$. **2.** $5\sqrt{2}$. **3.** Bod P. **4.** $|AB| = 14$. **5.** $|AB| = 5$, $|AC| = 8\sqrt{2}$, $|BC| = 13$. **7.** $\left[-\frac{11}{10}, 0, 0\right]$.

1.3

1. a) $\boldsymbol{AB} = (-1, -1, 6)$; b) $\boldsymbol{AB} = (4, -4)$; c) $\boldsymbol{AB} = (-1, 0, 1)$; d) $\boldsymbol{AB} = (7)$. **2.** a) Nie je, lebo $\boldsymbol{AB} = (1, 2, 1)$; b) je.

1.4

1. $|\boldsymbol{AB}| = \sqrt{29}$, $|\boldsymbol{AC}| = 3$, $|\boldsymbol{AB}| > |\boldsymbol{AC}|$. **2.** $u_3 = \pm 3\sqrt{6}$. **3.** $a_1 = \pm 3$.

1.5

1. $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = (0, 0)$. **2.** $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = (3, 5, 2)$. **3.** $[2, 7, 6]$.

1.6

1. $-\boldsymbol{i} = (-1, 0)$, $-\boldsymbol{j} = (0, -1)$. **2.** $\boldsymbol{a} = (-2, 1, 4)$.

1.7

1. a) $\boldsymbol{c} = (-1, -5)$; b) $\boldsymbol{c} = (-3, 4)$; c) $\boldsymbol{c} = (-1, 1)$.
2. a) $\boldsymbol{c} = (5, -2, -7)$; b) $\boldsymbol{c} = (0, 1, 0)$; c) $\boldsymbol{c} = (7, 7, 7)$.

1.9

1. a) – c) sú lineárne nezávislé. **2.** a) – c) Sú lineárne nezávislé.
3. $u_2 = 4$, $\boldsymbol{u} = -2\boldsymbol{v}$. **4.** $a_1 = 0$, $\boldsymbol{a} = 3\boldsymbol{b} - 6\boldsymbol{c}$.

1.10

1. a) 90°, vektory sú kolmé; b) 45°. **2.** 36°48′. **3.** $|AB| = 3\sqrt{3}$, $|AC| = 3\sqrt{2}$, $\varphi = 24°42'$. **4.** a) 67°23′, b) 45°.

1.11

1. $\boldsymbol{a} \cdot \boldsymbol{b} = -19$. **2.** a) $\boldsymbol{u} \cdot \boldsymbol{v} = 5$; b) $\boldsymbol{u} \cdot \boldsymbol{v} = 5\sqrt{2}$; c) $\boldsymbol{u} \cdot \boldsymbol{v} = -5$.
3. Najmenšia hodnota je -1, najväčšia hodnota je 1.
4. a) $|\boldsymbol{a}| = 9$; b) $|\boldsymbol{AB}| = 3\sqrt{2}$.

1.12

1. $\boldsymbol{AC} = (-4, 0, 6)$, $\boldsymbol{BD} = (9, -3, 6)$, $\boldsymbol{AC} \cdot \boldsymbol{BD} = 0$. **2.** $|\boldsymbol{AB}| = \sqrt{10}$, $|\boldsymbol{BC}| = \sqrt{50}$, $|\boldsymbol{AB}| = \sqrt{40}$, $|\boldsymbol{BC}|^2 = |\boldsymbol{AC}|^2 + |\boldsymbol{AB}|^2$. **3.** $|\boldsymbol{AD}| = |\boldsymbol{BC}|$, $|\boldsymbol{AB}| = |\boldsymbol{DC}|$, $\boldsymbol{AC} \cdot \boldsymbol{BD} = 0$, $|\boldsymbol{AC}| = |\boldsymbol{BD}| = 3\sqrt{2}$. **4.** Vektor $\boldsymbol{u} = (-3, 2)$, vektor $\boldsymbol{u}' = (3, -2)$ je k nemu opačný.

1.13

1. a) $x = 7 + 7t$, $y = 1 + 2t$; b) $x = -3t$, $y = 2$; c) $x = -2$, $y = -5 + 4t$; d) $x = 2t$, $y = 0$. **2.** $x = 3 - 2t$, $y = 3 - 5t$.
3. Na priamke ležia body B, C, neležia na nej body A, D, E.
4. Strany $x = -2t$, $y = 4 - 3t$; $x = 2 - 3s$, $y = 7 + 6s$; $x = 5 + 5r$, $y = 1 - 3r$; ťažnice $x = -3{,}5t$, $y = 4$; $x = 2 - 0{,}5s$, $y = 7 + 4{,}5s$; $x = 5 + 4r$, $y = 1 - 4{,}5r$.

1.14

1. $y_A = -\frac{3}{2}$, $y_B = -2$, $y_C = 1$, $y_D = -\frac{5}{2}$. **2.** $x_E = 2$, $x_F = \frac{7}{2}$, $x_G = \frac{13}{2}$, $x_H = -1$. **3.** a) $y = -2$; b) $x = 4$. **4.** $x - 3y - 2 = 0$.
5. a) $x - 3y + 16 = 0$; b) $2x + 3y - 3 = 0$; c) $x + 3y - 4 = 0$.
6. a) $x = 3 + 4t$, $y = 4 + 5t$; $5x - 4y + 1 = 0$; b) $x = -1 + 3t$, $y = -1 + 2t$; $2x - 3y - 1 = 0$; c) napr. $x = 2 + t$, $y = -1 - t$; $x + y - 1 = 0$; d) napr. $x = -7 + 10t$, $y = 8 - 10t$; $x + y - 1 = 0$; e) napr. $x = 3$, $y = 9 - 24t$, $x - 3 = 0$; f) napr. $x = 15t$, $y = 3 - 6t$; $2x + 5y - 15 = 0$.

1.15

1. a) $y = \frac{\sqrt{3}}{3}x + 10 - 5\frac{\sqrt{3}}{3}$; b) $y = -\sqrt{3}x + 4\sqrt{3}$;
c) $y = \sqrt{3}x + 5 - 3\sqrt{3}$. **2.** $y = -\frac{2}{3}x + 4$. **3.** $y_A = 3$. **4.** Priesečník s osou x má súradnice $[2, 0]$, s osou y má súradnice $\left[0, -\frac{7}{2}\right]$.

5. a) $k = \frac{7}{4}$; b) $k = \frac{3}{2}$; c) $k = \frac{5}{11}$. **6.** Na priamke ležia body A, B, neležia na nej body C, D. **7.** a) $y = \frac{1}{7}x + \frac{17}{7}$; b) $y = \frac{2}{5}x + 2$; c) $y = 3x - 1$.

8. a) $m = -\frac{5}{2}$; b) $m = -1$.

1.16

1. a) $[-1, 3]$; b) $\left[-\frac{5}{2}, -\frac{11}{2}\right]$. **2.** a) $\left[\frac{73}{48}, \frac{31}{48}\right], \left[\frac{93}{25}, \frac{24}{25}\right], \left[\frac{18}{17}, -\frac{44}{17}\right]$; b) $[4, 2]$, $[3, 1]$, $[5, 8]$. **3.** a) Splývajúce, keď $a = -\frac{5}{2}$, $b = -8$, rovnobežné rôzne, keď $a = -\frac{5}{2}$, $b \neq -8$, rôznobežné, keď $a \neq -\frac{5}{2}$; b) splývajúce, keď $a = 2\sqrt{3}$, $b = -4\sqrt{3}$ alebo $a = -2\sqrt{3}$, $b = 4\sqrt{3}$, rovnobežné rôzne, keď $a = 2\sqrt{3}$, $b \neq -4\sqrt{3}$ alebo $a = -2\sqrt{3}$, $b \neq 4\sqrt{3}$, rôznobežné, keď $a \neq \pm 2\sqrt{3}$. **4.** $a = \frac{k}{1+k}$, $b = \frac{k}{2(1+k)}$, $c = \frac{1}{1+k}$, $k \neq -1$. **5.** $B\left[\frac{7}{13}, \frac{22}{13}\right]$, $C\left[-\frac{2}{13}, \frac{16}{13}\right]$, $D\left[\frac{30}{13}, \frac{38}{5}\right]$; AB: $2x - 3y - 12 = 0$, AD: $3x + 2y - 5 = 0$.

1.17

1. a) 45°; b) 45°; c) 90°. **2.** a) $2x + y + 4 = 0$; b) $x - 2y + 7 = 0$. **3.** a) Rovnobežná $7x - 11y + 5 = 0$, kolmá $11x + 7y - 58 = 0$; b) rovnobežná $5x + 3y - 29 = 0$, kolmá $3x - 5y + 3 = 0$; c) rovnobežná $x + 2y - 10 = 0$, kolmá $2x - y - 5 = 0$; d) rovnobežná $2x + 3y - 17 = 0$, kolmá $3x - 2y - 6 = 0$. **4.** a) $\alpha = 61°16'$, $\beta = 73°44'$, $\gamma = 45°$; b) $\alpha = 26°34'$, $\beta = 63°26'$, $\gamma = 90°$. **5.** a) $P\left[\frac{83}{25}, \frac{119}{25}\right]$; b) $P[-5, 1]$. **6.** $5x - 4y - 7 = 0$, $4x - 5y - 8 = 0$.

1.18

1. $v_A = \frac{4\sqrt{5}}{5}$, $v_B = \frac{12\sqrt{5}}{5}$, $v_C = \frac{11\sqrt{5}}{5}$, $v_D = \frac{17\sqrt{5}}{5}$. **2.** $v_a = 2\sqrt{5}$,

$v_b = 2\sqrt{10}$, $v_c = 2\sqrt{10}$. **3.** a) $\frac{\sqrt{13}}{13}$; b) $\sqrt{5}$; c) $\frac{7\sqrt{17}}{17}$. **4.** $M_1\left[\frac{10}{3}, 2\right]$, $M_2\left[-\frac{34}{21}, -\frac{88}{63}\right]$, $M_3\left[\frac{160}{63}, \frac{82}{21}\right]$, $M_4\left[-\frac{152}{63}, \frac{4}{21}\right]$. **5.** $S = \frac{289}{130}$. **6.** a) $2x + 2y + 7 = 0$, $4x - 4y + 13 = 0$; b) $42x + 154y - 123 = 0$, $198x - 5y + 163 = 0$. **7.** $\frac{15}{2}$.

1.19

1. $x = 1 + 2t$, $y = 1 + t$, $z = -2 + 3t$. **2.** $x = -1 - 2t$, $y = 3 + 3t$, $z = 4 + t$. **3.** $x = 2 + 4t$, $y = 4 - t$, $z = -3 + 10t$. **4.** $y_1 = -3$, $z_1 = \frac{18}{5}$. **5.** AB: $x = 5 + 12t$, $y = 7 + 18t$, $z = 2 - 4t$, AC: $x = 5 + 10t$, $y = 7 + 4t$, $z = 2$, BC: $x = -7 - 2t$, $y = -11 - 14t$, $z = 6 + 4t$.

1.20

1.,2. a **4.** Priamky sú mimobežné. **3.** Priamky sú rovnobežné rôzne. **5.** Priamky sú rôznobežné.

1.21

1. $x = 1 - t - s$, $y = t$, $z = -s$. **2.** $x = 2 + t - 2s$, $y = -4 + 3t - 6s$, $z = 5 - t + 2s$, **3.** $x = -2 + 2t - 2s$, $y = 1 + 2t + 3s$, $z = 3 + 3t$. **4.** $x = 2t + s$, $y = t - 2s$, $z = 2t + 3s$.

1.22

1. $2x - 4y - 3z - 3 = 0$. **2.** $\boldsymbol{n} = (3; -4; 12)$. **3.** V rovine ležia body B a C, neleží v nej bod A. **4.** a) $16x - 3y - 4z - 37 = 0$; b) body A, B, C ležia na priamke, rovina nie je jednoznačne určená; c) $x - y = 0$. **5.** ϱ: $7x + 10y - z - 28 = 0$, σ: $5x + 4y - 2z - 42 = 0$, τ: $x + y - z - 1 = 0$. **6.** $2x + 6y + 5z - 26 = 0$. **7.** $4x - 7y + 4z - 81 = 0$.

1.23

1. $y + 5 = 0$. **2.** $x + 3y = 0$. **3.** $9y - z - 2 = 0$. **4.** a) Rovina je rovnobežná s osou z; b) v rovine leží os x; c) v rovine leží začiatok sústavy súradníc; d) v rovine leží os x; e) je to rovnica roviny yz; f) nie je to osobitný prípad. **5.** $2x + 6y + 5z - 26 = 0$. **6.** $2x + 3y + z - 11 = 0$.