Substitučná metóda

Riešenie:

1.) $\int (5x-1)^3 dx = \int t^3 \frac{dt}{5} = \frac{1}{5}\int t^3 dt = \frac{1}{5}\frac{t^4}{4} = \frac{1}{20}(5x-1)^4 + C$ $\quad t = 5x-1$

$\frac{dt}{dx} = 5 \Rightarrow dx = \frac{dt}{5}$

2.) $\int \frac{5x}{(x^2+4)^3} dx = \int \frac{5x}{t^3}\frac{dt}{2x} = \frac{5}{2}\int \frac{1}{t^3} dt = \frac{5}{2}\int t^{-3} dt =$ $\quad t = x^2+4$

$= \frac{5}{2}\frac{t^{-2}}{(-2)} = -\frac{5}{4t^2} = C - \frac{5}{4(x^2+4)^2}$ $\quad \frac{dt}{dx} = 2x \Rightarrow dx = \frac{dt}{2x}$

3.) $\int \sqrt[3]{4x-7}\, dx = \int \sqrt[3]{t}\frac{dt}{4} = \frac{1}{4}\int t^{\frac{1}{3}} dt = \frac{1}{4}\frac{t^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3}{16}t^{\frac{4}{3}} =$ $\quad t = 4x-7$

$= \frac{3}{16}\sqrt[3]{t^4} = \frac{3}{16}\sqrt[3]{(4x-7)^4} + C$ $\quad \frac{dt}{dx} = 4 \Rightarrow dx = \frac{dt}{4}$

4.) $\int e^{5x} dx = \int e^t \frac{dt}{5} = \frac{1}{5}\int e^t dt = \frac{1}{5}e^t = \frac{1}{5}e^{5x} + C,$ $\quad t = 5x$

$= \frac{1}{5}e^{5x} + C,$ $\quad \frac{dt}{dx} = 5 \Rightarrow dx = \frac{dt}{5}$

5.) $\int e^{1+\sin x}\cos x\, dx = \int e^{t}\cos x.\frac{dt}{\cos x} = \int e^t dt = e^t + C =$ $\quad t = 1+\sin x$

$= e^{1+\sin x} + C$ $\quad \frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

6.) $\int x.e^{x^2} dx = \int x.e^t \frac{dt}{2x} = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + C =$ $\quad t = x^2$

$= \frac{1}{2}e^{x^2} + C$ $\quad \frac{dt}{dx} = 2x \Rightarrow dx = \frac{dt}{2x}$

7.) $\int e^{3-2x}dx = \int e^t \frac{dt}{-2} = -\frac{1}{2}\int e^t dt = -\frac{1}{2}e^t + C = -\frac{1}{2}e^{3-2x} + C$ $\qquad t = 3-2x$

$= -\frac{1}{2}e^{3-2x} + C$ $\qquad \frac{dt}{dx} = -2 \Rightarrow dx = -\frac{dt}{2}$

8.) $\int 3.e^x\sqrt{1+e^x}dx = 3\int e^x\sqrt{t}\frac{dt}{e^x} = 3\int \sqrt{t}dt = 3\int t^{\frac{1}{2}}dt =$ $\qquad t = 1+e^x$

$= 3.\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = 2.t^{\frac{3}{2}} + C = 2\sqrt{(1+e^x)^3} + C$ $\qquad \frac{dt}{dx} = e^x \Rightarrow dx = \frac{dt}{e^x}$

$= 2\sqrt{(1+e^x)^3} + C$

9.) $\int \frac{e^{\frac{1}{x}}}{x^2}dx = \int \frac{e^t}{x^2}(-x^2)dt = -\int e^t dt = -e^t + C = C - e^{\frac{1}{x}}$ $\qquad t = \frac{1}{x}$

$= C - e^{\frac{1}{x}}$ $\qquad \frac{dt}{dx} = -\frac{1}{x^2} \Rightarrow dx = -x^2dt$

10.) $\int \frac{\sqrt{1+\ln x}}{x}dx = \int \frac{\sqrt{t}}{x}.x.dt = \int t^{\frac{1}{2}}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{t^3} + C =$ $\qquad t = 1+\ln x$

$= \frac{2}{3}\sqrt{(1+\ln x)^3} + C,$ $\qquad \frac{dt}{dx} = \frac{1}{x} \Rightarrow dx = x.dt$

11.) $\int \frac{1}{x\sqrt{\ln x}}dx = \int \frac{1}{x\sqrt{t}}.x.dt = \int \frac{1}{\sqrt{t}}dt = \int t^{-\frac{1}{2}}dt =$ $\qquad t = \ln x$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{t} + C = 2\sqrt{\ln x} + C,$ $\qquad \frac{dt}{dx} = \frac{1}{x} \Rightarrow dx = x.dt$

12.) $\int \frac{\ln^4 x}{x}dx = \int \frac{t^4}{x}.x.dt = \int t^4 dt = \frac{t^5}{5} + C = \frac{1}{5}\ln^5 x + C =$ $\qquad t = \ln x$

$= \frac{1}{5}\ln^5 x + C,$ $\qquad \frac{dt}{dx} = \frac{1}{x} \Rightarrow dx = x.dt$

13.) $\int \cos\frac{x}{4}dx = \int \cos t.4dt = 4\int \cos t.dt = 4\sin t + C =$ $\qquad t = \frac{x}{4}$

$= 4\sin\frac{x}{4} + C$ $\qquad \frac{dt}{dx} = \frac{1}{4} \Rightarrow dx = 4.dt$

14.) $\int \sin 2x dx = \int \sin t\frac{dt}{2} = \frac{1}{2}\int \sin t dt = -\frac{1}{2}\cos t + C =$ $\qquad t = 2x$

$= -\frac{1}{2}\cos 2x + C,$ $\qquad \frac{dt}{dx} = 2 \Rightarrow dx = \frac{dt}{2}$

15.) $\int \cot g(2x+1)dx = \int \cot g t.\frac{dt}{2} = \frac{1}{2}\int \cot g t dt =$ $\qquad t = 2x+1$

$= \frac{1}{2}\int \frac{\cos t}{\sin t}dt = \frac{1}{2}.\ln t + C = \frac{1}{2}.\ln|\sin t| + C = \frac{1}{2}\ln|\sin(2x+1)| + C =$ $\qquad \frac{dt}{dx} = 2 \Rightarrow dx = \frac{dt}{2}$

$= \frac{1}{2}\ln|\sin(2x+1)| + C,$

19.) $\int \frac{\sqrt[3]{tg^2 x}}{\cos^2 x}dx = \int \frac{\sqrt[3]{t^2}}{\cos^2 x}.dt.\cos^2 x = \int t^{\frac{2}{3}}dt = \frac{t^{\frac{5}{3}}}{\frac{5}{3}} =$ $\qquad t = tg\, x$

$= \frac{3}{5}t^{\frac{5}{3}} + C = \frac{3}{5}\sqrt[3]{t^5} + C = \frac{3}{5}\sqrt[3]{tg^5 x} + C,$ $\qquad \frac{dt}{dx} = \frac{1}{\cos^2 x} \Rightarrow dx = dt.\cos^2 x$

20.) $\int \frac{1}{\cos^2 x.\sqrt{5+tgx}}dx = \int \frac{1}{\cos^2 x.\sqrt{t}}dt.\cos^2 x =$ $\qquad t = 5+tgx$

$= \int \frac{1}{\sqrt{t}}dt = \int t^{-\frac{1}{2}}dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{5+tg} + C,$ $\qquad \frac{dt}{dx} = \frac{1}{\cos^2 x} \Rightarrow dx = dt.\cos^2 x$

21.) $\int \frac{2x^2}{\cos^2(x^3+1)}dx = \int \frac{2x^2}{\cos^2 t}.\frac{dt}{3x^2} = \frac{2}{3}\int \frac{1}{\cos^2 t}dt =$ $\qquad t = x^3 + 1$

$= \frac{2}{3}.tg t + C = \frac{2}{3}tg(x^3+1) + C,$ $\qquad \frac{dt}{dx} = 3x^2 \Rightarrow dx = \frac{dt}{3x^2}$

22.) $\int \sin x . \cos x \, dx = \int t . \cos x . \frac{dt}{\cos x} = \int t . dt = \frac{t^2}{2} + C$ $\qquad\qquad$ $\neq \sin x$

$= \frac{1}{2} \sin^2 x + C,$ $\qquad\qquad$ $\frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

23.) $\int \sin^3 x . \cos x \, dx = \int t^3 \cos x \frac{dt}{\cos x} = \int t^3 dt = \frac{t^4}{4} + C =$ $\qquad\qquad$ $t = \sin x$

$= \frac{1}{4} \sin^4 x + C,$ $\qquad\qquad$ $\frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

24.) $\int \sin^2 x . \cos^2 x \, dx = \int \left(\frac{\sin 2x}{2} \right)^2 dx = \frac{1}{4} \int \sin^2 2x =$ $\qquad\qquad$ $t = 2x$

$= \frac{1}{4} \int \sin^2 t . \frac{dt}{2} = \frac{1}{8} \int \sin^2 t \, dt = \frac{1}{8} \left[\frac{t}{2} - \frac{1}{4} \sin 2t \right] + C =$ $\qquad\qquad$ $\frac{dt}{dx} = 2 \Rightarrow dx = \frac{dt}{2}$

$= \frac{1}{8} \left[\frac{2x}{2} - \frac{1}{4} \sin (2.2x) \right] + C = \frac{1}{8} \left[x - \frac{1}{4} \sin 4x \right] + C =$

$= \frac{1}{8} x - \frac{1}{32} . \sin 4x + C$