Integrál - per partes

1 Ako sa používa metóda Per partes?

$$(u.v)' = u'.v + u.v'$$

$$u'.v = (u.v)' - u.v'$$

$$\int u'.v\,dx = u.v - \int u.v'\,dx, \qquad \vee \qquad \int u.v'\,dx = u.v - \int u'.x\,dx$$

$$\int x.e^x\,dx =$$

$$\int x.e^x\,dx = x.e^x - \int e^x.1dx = x.e^x - e^e + C = e^x(x-1) + C, \qquad u' = e^x, \quad u = e^x$$

$$v = x, \qquad v' = 1$$

2 Riešenie:

2.) $$\int \frac{x}{3}.e^x\,dx = \frac{1}{3}\int xe^x\,dx = \frac{1}{3}\left[x.e^x - \int 1.e^x dx\right] = \frac{1}{3}\left[x.e^x - e^x\right] + C = \qquad u = x, \quad u' = 1$$

$$= \frac{1}{3}.e^x(x-1) + C, \qquad v' = e^x, \quad v = e^x$$

3.) $$\int \ln x\,dx = \int 1.\ln x\,dx = x.\ln x - \int \frac{1}{x}.x\,dx = x.\ln x - \int 1.dx = \qquad u = \ln x, \quad u' = \frac{1}{x}$$

$$= x.\ln x - x + C = x(\ln x - 1) + C \qquad v' = 1, \quad v = x$$

4.) $$\int x.\sin x\,dx = -x.\cos x - \int 1.(-\cos x)dx = -x.\cos x + \int \cos x\,dx = \qquad u = x, \quad u' = 1$$

$$= -x.\cos x + \sin x + C \qquad v' = \sin x, \; v = -\cos x$$

5.) $\int x^3 \ln x\,dx = \frac{x^4}{4}\ln x - \int \frac{1}{x}.\frac{x^4}{4}dx = \frac{x^4}{4.}.\ln x - \frac{1}{4}\int x^3 dx =$ $\quad u = \ln x, \quad u' = \frac{1}{x}$

$= \frac{x^4}{4}.\ln x - \frac{1}{4}.\frac{x^4}{4} + C = \frac{x^4}{16}(4\ln x - 1) + C,$ $\quad v' = x^3, \quad v = \frac{x^4}{4}$

6.) $\int x^2 e^x dx = x^2 e^x - \int 2xe^x dx = x^2 e^x - 2\int xe^x dx =$ $\quad u = x^2, \quad u' = 2x$

$= x^2 e^x - 2e^x(x-1) + C = x^2 e^x - 2xe^x + 2e^x + C =$ $\quad v' = e^x, \quad v = e^x$

$= e^x(x^2 - 2x + 2) + C,$

7.) $\int x^2.\ln x\,dx = \frac{x^3}{3}\ln x - \int \frac{1}{x}\frac{x^3}{3}dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx =$ $\quad u = \ln x, \quad u' = \frac{1}{x}$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3} + C = \frac{x^3}{9}(3\ln x - 1) + C$ $\quad v' = x^2, \quad v = \frac{x^3}{3}$

8.) $\int x.\cos x\,dx = x.\sin x - \int 1.\sin x\,dx =$ $\quad u = x, \quad u' = 1$

$= \underline{x.\sin x + \cos x + C}$ $\quad v = \sin x, \quad v' = \cos x$

9.) $\int \frac{\ln x}{x}dx = \int \frac{1}{x}\ln x\,dx = \ln x.\ln x - \int \frac{\ln x}{x}dx$ $\quad u = \ln x, \; u' = \frac{1}{x}$

$2\int \frac{\ln x}{x}dx = \ln^2 x + C$ $\quad v' = \frac{1}{x} \quad v = \ln x$

$\underline{\int \frac{\ln x}{x}dx = \frac{1}{2}\ln^2 x + C}$

10.) $\int x.\ln x\,dx = \frac{x^2}{2}\cdot\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}dx =$ $\quad u' = x, \quad u = \frac{x^2}{2}$

$= \frac{x^2 \ln x}{2} - \frac{1}{2}\int x\,dx = \frac{x^2 \ln x}{2} - \frac{1}{2}\cdot\frac{x^2}{2} + C$ $\quad v = \ln x, \; v' = \frac{1}{x}$

$= \underline{\frac{x^2 \ln x}{2} - \frac{x^2}{4} + C}$

11.) $\int \sin x . \cos x \, dx = \sin x . \sin x - \int \sin x . \cos x \, dx \qquad u' = \cos x, \quad u = \sin x$

$\int \sin x . \cos x \, dx = \sin^2 x - \int \sin x . \cos x \, dx \qquad v = \sin x, \quad v' = \cos x$

$2\int \sin x . \cos x \, dx = \sin^2 x$

$\int \sin x . \cos x \, dx = \frac{\sin^2 x}{2} + C$

12.) $\int \cos^2 x \, dx = \sin x . \cos x - \int \sin x (- \sin x) dx \qquad u' = \cos x, \quad u = \sin x$

$\int \cos^2 x \, dx = \sin x . \cos x + \int \sin^2 x \, dx \qquad v = \cos x, \quad v' = -\sin x$

$\int \cos^2 x \, dx = \sin x . \cos x + \int (1 - \cos^2 x) dx$

$\int \cos^2 x \, dx = \sin x . \cos x + \int dx - \int \cos^2 x \, dx$

$2\int \cos^2 x \, dx = \sin x . \cos x + x$

$\int \cos^2 x \, dx = \frac{1}{2}(x + \sin x . \cos x) + C$

13.) $\int x . e^{2x} dx = \frac{1}{2} x . e^{2x} - \int \frac{1}{2} e^{2x} dx \qquad u' = e^{2x}, \quad u = \frac{1}{2} e^{2x}$

$\int x . e^{2x} dx = \frac{1}{2} x . e^{2x} - \frac{1}{2} \cdot \frac{1}{2} e^{2x} \qquad v = x, \quad v' = 1$

$\int x . e^{2x} dx = \frac{1}{2} x . e^{2x} - \frac{1}{4} e^{2x}$

$\int x . e^{2x} dx = \frac{1}{2} e^{2x} \left(x - \frac{1}{2} \right) + C$
